Bayesian Multi-Object Filtering with Amplitude Feature Likelihood for Unknown Object SNR

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***Abstract*—In many tracking scenarios, the amplitude of target returns are stronger than those coming from false alarms. This information can be used to improve the multi-target state estimation by obtaining more accurate target and false alarm likelihoods. Target amplitude feature is well known to improve data association in conventional tracking filters, such as the Probabilistic Data Association (PDA) and Multiple Hypothesis Tracking (MHT), and results in better tracking performance of low SNR targets. The advantage of using the target amplitude approach is that targets can be identified earlier through the enhanced discrimination between target and false alarms. One of the limitations of this approach is that it is usually assumed that the SNR of the target is known. We show that the reliable estimation of the SNR requires a significant number of measurements and so we propose an alternative approach for situations where the SNR is unknown. We illustrate this approach in the context of multiple targets for different signal to noise ratios in the framework of Finite Set Statistics (FISST). Furthermore, we illustrate how this can be incorporated into approximate multi-object filters derived from FISST, including Probability Hypothesis Density (PHD) and Cardinalized PHD (CPHD) filters. We present simulation results for Gaussian mixture implementations of the filters that demonstrate a significant improvement in performance over just using location measurements.**

Keywords: Tracking, PHD filters, Finite Set Statistics, random sets, Bayesian filtering, target amplitude feature, multi-object estimation.

I. Introduction

In multi-target tracking, a set of measurements is given to the filter, from which the aim is to estimate a set of target states. Due to the nature of the sensor, a detector providing the measurements of the targets to the tracker is rarely perfect: some of the measurements may be false alarms, while the true targets may not be detected at all. The tracker needs to identify the correct targets from the measurements using the observation model which describes how these relate to the target states and track them based on the state model which describes the motion of the targets. In many sensors, such as radar and sonar, the measurements of the targets' positions would also come with a signal strength, or amplitude, of the signal. The strength of the signal from a target is typically stronger than the false alarms and so provides a valuable source of information to determine if the measurement is from a false alarm or from a true target. We investigate using this information in our measurement model for targets with a range of different signal-to-noise ratios (SNRs).

Using the target amplitude strength as a measurement in target tracking, in addition to the conventional measurements of range, azimuth, Doppler etc, can result in improved data association, reduced number of false tracks and better performance for low SNR targets. In this way the formation of tracks is based not only in the consistency of target motion (in the probabilistic sense) but also in the consistency of its amplitude returns (again in the probabilistic sense). The pioneers of utilization of target amplitude have been Colegrove [1] and Lerro & Bar-Shalom [2] in the context of single target tracking in clutter using the PDA filter. Subsequently the target amplitude has been incorporated in the MHT framework [3] and Viterbi data association scheme [4]. More recently the significance of target amplitude has been explored for data association of closely spaced targets in [5]. The utilization of target amplitude can be seen as a step towards the track-before-detect approach, where the entire intensity surface in the measurement space is treated as a measurement.

Whilst it has been shown that this results in improved performance, these techniques have relied on knowledge of the signal-to-noise ratio of the target. Using the Fisher Information and the Cramer-Rao Lower Bound, we show that estimating the SNR of a Rayleigh distributed target requires a large amount of data and hence it may not be possible to reliably estimate it in practice. Instead of this, we propose an alternative approach by constructing an amplitude likelihood for when the target SNR is unknown by marginalising a likelihood over a range of possible values. In our simulated results, we show that this still results in a significant improvement in estimation performance over not using the amplitude information.

In the paper we aim to exploit the target amplitude information in order to improve the estimates of filters derived from the Finite Set Statistics multi-object Bayes filter. Since the target amplitude measurement is fairly straightforward to incorporate into the filter and at the same time the amplitude information is readily available in most practical applications (radar, sonar, acoustics, etc), we aim to develop a more accurate multi-object filters with only a minor additional computational load. Our presentation is carried out in the context of radar or sonar data, where both the clutter and the target amplitude are modelled using the Rayleigh distribution (the parameter being the SNR). In other applications, other amplitude probabilistic models may be more appropriate, but the general conclusions of this study should be applicable in a wider context.

We show that an amplitude feature likelihood, both for known and unknown target SNR, can be incorporated into the generic multi-object Bayes filter and in the approximation strategies for using this in practice, including the Probability Hypothesis Density (PHD) filter [6] and the Cardinalized PHD (CPHD) filter [7]. We present results of this approach for the

Gaussian mixture implementations of the PHD and CPHD filters [8], [9] and demonstrate the improved performance achieved by using the amplitude information.

The paper is organised as follows. Section II presents a short review of the multi-object Bayesian filter. Section III demonstrates how the amplitude likelihood model (for known and unknown SNR) can be incorporated into the the multi-object Bayes filter and its approximations, the PHD filter and CPHD filter. Section IV illustrates the concept in the context of the Rayleigh fluctuating target amplitude. First the amplitude likelihood for known SNR is considered and then we show how the target SNR can be marginalized over a range of values to give the likelihood for unknown SNR. Section V presents an extensive set of simulation results which validate the approach. The conclusions of this study are given in Section VI.

II. Bayesian Multi-Object Filtering

Suppose that at time k there are $N(k)$ object states $x_{k,1},\ldots,x_{k,N(k)}$, each taking values in a state space $\mathcal{X}\subseteq\mathbb{R}^{n_x}$, and $M(k)$ measurements (detections) $z_{k,1},\ldots,z_{k,M(k)}$, each taking values in the observation space $\mathcal{Z}\subseteq\mathbb{R}^{n_z}$. A multi-object state and a multi-object observation are then represented by random finite sets:

$$\begin{aligned} X_k &= \{x_{k,1},\ldots,x_{k,N(k)}\}\in\mathcal{F}(\mathcal{X}) \\ Z_k &= \{z_{k,1},\ldots,z_{k,M(k)}\}\in\mathcal{F}(\mathcal{Z}), \end{aligned} \tag{1}$$

where $\mathcal{F}(\mathcal{X})$ and $\mathcal{F}(\mathcal{Z})$ are the finite subsets of $\mathcal{X}$ and $\mathcal{Z}$ respectively. Due to the imperfections of detectors, it is possible that at time k some of the targets in X_k are not detected. Moreover, the observation set Z_k typically includes false detections (or clutter) in addition to target originated detections.

The objective of Mahler's Bayesian multi-object filtering [10] is to determine at each time step k the posterior probability density of multi-object state $f_{k|k}(X_k|Z_{1:k})$, where $Z_{1:k}=(Z_1,\ldots,Z_k)$ denotes the accumulated observations up to time k. The multi-object posterior can be computed sequentially via the prediction and update steps. Suppose that $f_{k|k}(X_k|Z_{1:k})$ is known and that a new set of measurements Z_{k+1} corresponding to time $k+1$ has been received. Then the predicted and updated multi-object posterior densities are calculated as follows [10]:

$$f_{k+1|k}(X|Z_{1:k})=\int f_{k+1|k}(X|X')f_{k|k}(X'|Z_{1:k})\delta X' \tag{2}$$

$$f_{k+1|k+1}(X|Z_{1:k+1})=\frac{f_{k+1|k}(Z_{k+1}|X)f_{k+1|k}(X|Z_{1:k})}{\int f_{k+1|k}(Z_{k+1}|X)f_{k+1|k}(X|Z_{1:k})\delta X}, \tag{3}$$

where $f_{k+1|k}(X|X')$ is a multi-object transition density and $f_{k+1|k}(Z_{k+1}|X)$ is a multi-object likelihood. We should note that this recursion is a non-trivial generalization, since the transition density needs to consider the uncertainty in target number, which can change over time due to targets entering and leaving the state space, and the multi-object likelihood needs to consider detection uncertainty and false alarms. It is also clear that the integrals in the recursion are non-standard and need the notion of a set integral, which is defined as a mixture of probability distributions over all different cardinalities (see [10]).

The Bayesian multi-object filter described here cannot be implemented in a computationally tractable manner. Among the approximations that were recently proposed of particular importance are the PHD filter the CPHD filter [10]. Instead of propagating the full multi-target density $f_{k|k}(X_k|Z_{1:k})$, the PHD filter propagates its first order moment, called the PHD or intensity function. The CPHD propagates not only the PHD but also the cardinality distribution (the probability distribution of the number of targets).

III. Bayesian Multi-Object Filtering with Amplitude Feature

In this section we show how the signal amplitude is introduced to the multi-object likelihood to enhance the discrimination between targets and false alarms with the multi-object Bayes filter. We describe the likelihood function using the amplitude and incorporate it into the multi-object Bayes filter, the PHD pseudo-likelihood and the CPHD pseudo-likelihood.

A. Amplitude feature likelihoods

We consider an augmented state that contains the positions, p_1 and p_2, their velocities, $\dot{p}_1$ and $\dot{p}_2$, $\tilde{x}=[p_1\ p_2\ \dot{p}_1\ \dot{p}_2]^T$, and parameter d, i.e.

$$x:=[\tilde{x}^T\ d]^T. \tag{4}$$

The expected (or mean) SNR is $1+d$, which is typically defined in the log scale as

$$\text{SNR(dB)}=10\log_{10}(1+d). \tag{5}$$

and is assumed to be constant for each target. The case where $d=0$ corresponds to the situation where the power of the target and false alarm signals are the same.

Each measurement consists of a two-dimensional target position $\tilde{z}$ and amplitude $a\geq 0$, that is, a measurement vector has the form $z:=[\tilde{z}^T\ a]^T$, with $n_z=3$.

We assume that the amplitude of the signal return is independent of target state location, and thus the likelihoods for targets $g(z|x)$ and the likelihood for clutter $c(z)$ are given by

$$g(z|x)=g_{\tilde{z}}(\tilde{z}|\tilde{x})g_a(a|d), \tag{6}$$

$$c(z)=c_{\tilde{z}}(\tilde{z})c_a(a). \tag{7}$$

Here $g_a(a|d)$ and $c_a(a)$ are the *amplitude* likelihood functions for target and clutter, respectively. The detection process typically consist of finding local maxima among all measurements followed by thresholding at a certain level $\tau>0$. If we denote the amplitude likelihoods for the measurements that exceed the detection threshold as $g_a^\tau(a|d)$ and $c_a^\tau(a)$, then we have (due to normalisation):

$$g_a(a|d) = g_a^\tau(a|d)p_D^\tau(d) \tag{8}$$

$$c_a(a) = c_a^\tau(a)p_{FA}^\tau \tag{9}$$

where

$$p_D^\tau(d) = \int_\tau^\infty g_a(a|d)da, \quad (10)$$

$$p_{FA}^\tau = \int_\tau^\infty c_a(a)da \quad (11)$$

are the probability of detection and the probability of false alarm, respectively.

B. Multi-Object Likelihood with Amplitude Information

For a given multi-target state $X = \{x_1, \ldots, x_n\}$, the random measurement set collected by the sensor is of the form

$$Z = \left(\bigcup_{i=1}^{n} \Sigma(x_i)\right) \cup \Theta, \quad (12)$$

where $\Sigma(x_i)$ is the Random Finite Set (RFS) generated by the single target state x_i, which either contains a single measurement z_i or is empty and takes on the value $\emptyset$, and Θ contains false alarms with expected number λ before thresholding [1].

Let λ define the expected number of false alarms before thresholding. Then the product λp_{FA}^τ gives the expected number of false alarms after thresholding, which is typically assumed to be Poisson distributed. Following the derivation in [10, ch.12] the probability density of clutter is

$$f_C(K) := \exp(-\lambda \cdot p_{FA}^\tau) \prod_{z \in K} (\lambda \cdot p_{FA}^\tau) c_a^\tau(d) c_{\tilde{z}}(\tilde{z}|\tilde{x}). \quad (13)$$

Incorporating the amplitude likelihoods into Mahler's multi-object likelihood (see [10]) we get

$$f_{k+1}(Z_{k+1}|X) = f_C(Z_{k+1}) \cdot \prod_{x \in X} (1 - p_D^\tau(d)) \times$$
$$\sum_{\theta} \prod_{i:\theta(i)>0} \frac{p_D^\tau(d_i) \cdot g_{\tilde{z}}(\tilde{z}_{\theta(i)}|\tilde{x}_i) \cdot g_a^\tau(a_{\theta(i)}|d_i)}{(1 - p_D^\tau(d_i)) \cdot \lambda \cdot p_{FA}^\tau \cdot c_{\tilde{z}}(\tilde{z}_{\theta(i)}) \cdot c_a^\tau(a_{\theta(i)})}, \quad (14)$$

where the sum is over all possible associations θ between X and Z, i.e. $\theta : \{1, \ldots, n\} \to \{0, 1, \ldots, M\}$.

Note here that the difference between the standard multi-object likelihood [10] with the standard measurement model and the likelihood with amplitude information in (14) is the introduction of a factor of

$$\frac{g_a^\tau(a_{\theta(i)}|d_i)}{c_a^\tau(a_{\theta(i)})}, \quad (15)$$

which Lerro and Bar-Shalom called the *amplitude likelihood ratio*. They used it to modify the standard PDAF association probabilities to include amplitude discrimination in [2].

[1] Typically in radar and sonar signal processing, all the local maxima are found in the raw data first: false alarms before thresholding refer to the local maxima.

C. PHD pseudo-likelihood

The amplitude information can also be incorporated into the PHD update, as described in [11]. Amplitude information has previously been incorporated into the sequential Monte Carlo implementation of the PHD filter [12]. The PHD pseudo-likelihood (see [10]) is given by

$$L_Z(x) = (1 - p_D^\tau(d)) \quad (16)$$
$$+ p_D^\tau(d) \sum_{z \in Z} \frac{p_D^\tau(d) g_a^\tau(a|d) g_{\tilde{z}}(\tilde{z}|\tilde{x})}{\lambda p_{FA}^\tau c_a^\tau(a) c_{\tilde{z}}(\tilde{z}) + \langle D_{k+1|k}, p_D^\tau(d) \cdot g_a^\tau(\cdot|d) g_{\tilde{z}} \rangle},$$

where $D_{k+1|k}$ is the predicted PHD. Multiplying this by the predicted PHD gives the PHD update.

D. CPHD pseudo-likelihood

The amplitude information can similarly be incorporated into the CPHD update, as described in the following. Denote by C_j^ℓ the *binomial coefficient* $\frac{\ell!}{j!(\ell-j)!}$, P_j^n the *permutation coefficient* $\frac{n!}{(n-j)!}$, and $e_j(\cdot)$ the *elementary symmetric function* of order j defined for a finite set Z of real numbers by

$$e_j(Z) = \sum_{S \subseteq Z, |S|=j} \left(\prod_{\zeta \in S} \zeta\right),$$

with $e_0(Z) = 1$ by convention.

The CPHD filter pseudo-likelihood (see [10, p.638] or [9, III-A]) is given by

$$L_Z(x) = (1 - p_D^\tau(d)) \frac{\langle \Upsilon_{k+1}^1[D_{k+1|k}; Z], \rho_{k+1|k} \rangle}{\langle \Upsilon_{k+1}^0[D_{k+1|k}; Z], \rho_{k+1|k} \rangle} \quad (17)$$
$$+ \sum_{z \in Z} \psi_{k+1,z}(x) \frac{\langle \Upsilon_{k+1}^1[D_{k+1|k}; Z - \{z\}], \rho_{k+1|k} \rangle}{\langle \Upsilon_{k+1}^0[D_{k+1|k}; Z], \rho_{k+1|k} \rangle},$$

where

$$\Upsilon_{k+1}^u[D, Z](n) = \sum_{j=0}^{\min(|Z|,n)} (|Z|-j)! \rho_{K,k+1}(|Z|-j) P_{j+u}^n \quad (18)$$
$$\times \frac{\langle 1 - p_D^\tau(d), D \rangle^{n-(j+u)}}{\langle 1, D \rangle^n} e_j(\Xi_k(D, Z)),$$

$$\psi_{k+1,z}(x) = [c_a^\tau(d) c_{\tilde{z}}(\tilde{z})]^{-1} g_a^\tau(a|d) g_{\tilde{z}}(\tilde{z}|\tilde{x}) p_D^\tau(d), \quad (19)$$

$$\Xi_{k+1}(D, Z) = \{\langle D, \psi_{k+1,z} \rangle : z \in Z\}, \quad (20)$$

$$\rho_{K,k+1}(n) = e^{-\lambda p_{FA}^\tau} \frac{(\lambda p_{FA}^\tau)^n}{n!}, \quad (21)$$

$D_{k+1|k}$ and $\rho_{k+1|k}$ are the predicted PHD and cardinality distribution respectively. Multiplying the CPHD pseudo-likelihood by the predicted PHD gives the CPHD update. The cardinality distribution update is given by the Bayes update

$$\rho_{k+1}(n) = \frac{\Upsilon_{k+1}^0[D_{k+1|k}; Z](n) \rho_{k+1|k}(n)}{\langle \Upsilon_{k+1}^0[D_{k+1|k}; Z], \rho_{k+1|k} \rangle},$$

where $\Upsilon_{k+1}^0[D_{k+1|k}; Z]$ is interpreted as the likelihood of the cardinality of the measurement set.

IV. AMPLITUDE FEATURE LIKELIHOOD MODEL

In order to illustrate the general concept of an amplitude enhanced multi-object Bayesian filter, in this section we consider the specific probability distributions for the target and false alarm amplitude measurements. The adopted amplitude models will determine the probababilities of target detection and false alarm for given thresholds. We first present the likelihoods for known SNR and then show how we can derive likelihoods for unknown SNR.

A. Rayleigh Target Likelihood

Most targets of interest in radar signal processing create radar returns which are composed of a sum of returns from individual scattering points which make up the target. As a result, the target SNR is typically fluctuating in a random manner, with four major fluctuation models used in practice (known as Swerling type 1, 2, 3, and 4) [13]. We adopt the Rayleigh target amplitude fluctuating model from Lerro and Bar-Shalom [2], where the amplitude a is assumed to come from the output of a bandpass matched filter followed by an envelope detector. This model is applicable to Swerling type 1 and 2 targets, and is a reasonable representative of aircraft magnitude fluctuations. The probability densities of amplitudes of false detections (clutter) and target are adopted as [13]:

$$p_0(a) := a \exp\left(\frac{-a^2}{2}\right), \qquad a \geq 0 \tag{22}$$

$$p_1^d(a) := \frac{a}{1+d} \exp\left(\frac{-a^2}{2(1+d)}\right), \qquad a \geq 0 \tag{23}$$

respectively, and hence

$$p_0^\tau(a) = a \exp\left(\frac{\tau^2 - a^2}{2}\right), \qquad a \geq \tau \tag{24}$$

$$p_1^{\tau,d}(a) = \frac{a}{1+d} \exp\left(\frac{\tau^2 - a^2}{2(1+d)}\right), \qquad a \geq \tau \tag{25}$$

These would be our amplitude likelihood functions for false alarms $c_a^\tau(a)$ and targets $g_a^\tau(a|d)$, that is

$$c_a^\tau(a) := p_0^\tau(a), \tag{26}$$

$$g_a^\tau(a|d) := p_1^{\tau,d}(a). \tag{27}$$

Let us consider a sample of n random variables $a_1, a_2, a_3, \ldots, a_n$ drawn i.i.d. from the likelihood function $g_a(a|d)$. Then we have

$$g_a(a_1, a_2, a_3, \ldots, a_n|d) = \prod_{i=1}^{n} g_a(a_i|d). \tag{28}$$

The variance of any unbiased estimator $\hat{d}$ is bounded by the inverse of the Fisher Information [14], $I(d)$,

$$var(\hat{d}) \geq \frac{1}{I(d)}, \tag{29}$$

where

$$I(d) = \mathrm{E}_d\left[\left(\frac{\partial(ln(\prod_{i=1}^{n} g_a(a_i|d)))}{\partial d}\right)^2\right]. \tag{30}$$

The inverse of the Fisher Information is known as the Cramer-Rao Lower Bound (CRLB). It can be shown that if the likelihood function (28) is a product of Rayleigh distributions, then the Fisher information is

$$I(d) = \frac{n}{(1+d)^2}, \tag{31}$$

and hence the CRLB is

$$\frac{(1+d)^2}{n.} \tag{32}$$

For a target with the mean SNR $1+d$, for a 20% error to be within 2 standard deviations, we need

$$2\sqrt{\frac{(1+d)^2}{n}} < 0.2(1+d), \tag{33}$$

which leads to

$$n > \frac{(1+d)^2}{0.01(1+d)^2} = 100 \tag{34}$$

irrespective of the value of d.

The significance of (34) is that if we do not know the SNR of a target in advance and want to estimate it, we require a large number of measurements from the target, which is not practical for most tracking applications. Note that in the above we considered the CRLB for a single target case where we know the correct measurement association. In practice, the associations would be unknown and hence the error would be even higher than in equation (32). Therefore instead of attempting to estimate d, in this section we consider two cases, where either the SNR is known or, if the SNR is unknown, it is integrated out from the target amplitude likelihood and therefore not included in the state vector. Hence in both cases we effectively consider the target state $x \equiv \tilde{x}$.

B. Known SNR

When the target SNR d is known, we can use $g_a^\tau(a|d)$ from (27) as our target amplitude likelihood function.

Since each target in the surveillance region may have a different value of mean SNR $1+d$, typically the detection threshold τ is computed for a specified value of p_{FA}^τ. The analytic expressions for p_{FA}^τ and $p_D^\tau(d)$ follow from (22) and (23). Table I lists the values of p_D^τ for different SNR values d and specified p_{FA}^τ. Figure 1 shows the amplitude likelihoods for clutter and $10dB$, $20dB$ and $30dB$ targets.

Table I
TARGET p_D UNDER DIFFERENT SNR AND p_{FA} COMBINATIONS

	SNR	10 dB	15 dB	20 dB	25dB	30 dB
p_{FA}	τ					
0.001	3.7169	0.5337	0.8092	0.9339	0.9785	0.9931
0.01	3.0349	0.6579	0.8683	0.9554	0.9856	0.9954
0.05	2.4477	0.7616	0.9123	0.9708	0.9906	0.9970
0.1	2.1460	0.8111	0.9319	0.9775	0.9928	0.9977

We are unlikely to know the true SNR in practice and, as we showed in the previous section, we would require a large number of observations in order to estimate it. Hence, a

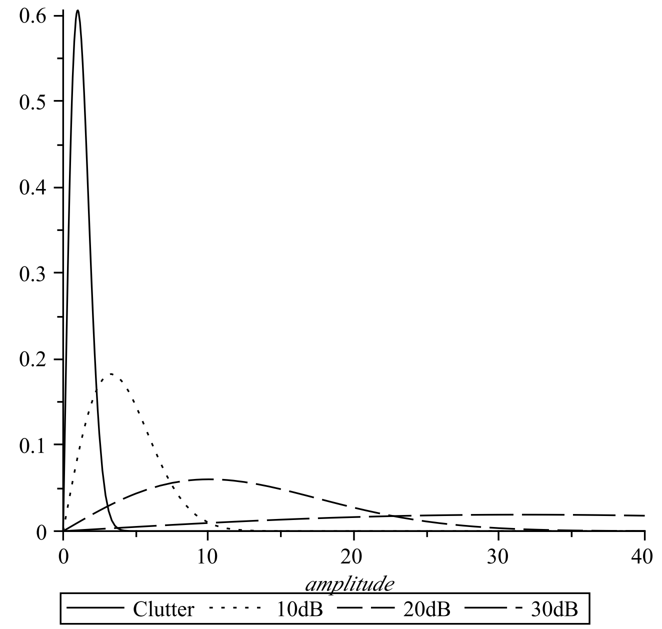

Figure 1. Amplitude pdfs for Rayleigh likelihoods: clutter and known SNR

typical approach would be for the engineer to provide a value for what the SNR should be and hope that this adequately characterises the target. If we assume that the target SNR is fixed, then the effect of providing an incorrect amplitude likelihood function can be measured with the Kullback-Leibler divergence between the amplitude likelihood with true SNR d and likelihood with chosen SNR $\hat{d}$, which can be calculated with

$$D(g_a(a|d)||g_a(a|\hat{d})) := \int_{a=0}^{\infty} g_a(a|d) \ln \left(\frac{g_a(a|d)}{g_a(a|\hat{d})} \right) da, \tag{35}$$

measured in nats (information units in base e) [14].

Table II shows the effect of choosing an incorrect value for a range of estimated and true SNR combinations. A low value indicates that there is not much difference in the distributions and hence is a good choice (note that the diagonal is zero since it is the same as the true distribution). For example, the first column shows how effective the choice is when we assume that the target SNR is $10dB$, which is a very poor approximation for a $30dB$ target ($94nats$).

For any particular choice of SNR, we will not accurately characterise the full range of possible true target SNRs. In fact, the values in table II assume that the threshold is zero. The actual values will depend on the threshold which further compounds the error since it will give an incorrect probability of detection. Hence if we assume that there is a high SNR but the actual target has low SNR, we will threshold correct observations, whereas if the target has a high SNR but we assume it is low then we will set the threshold too low and process a larger number of obervations than necessary.

In the next section, we show that by marginalising over a range of SNR values, we can more accurately characterise a larger range of target SNRs.

Table II
KULLBACK-LEIBLER DIVERGENCE BETWEEN TRUE AND ESTIMATED AMPLITUDE DISTRIBUTIONS (IN NATS)

Estimated	10 dB	15 dB	20 dB	25dB	30 dB
True					
10dB	0	0.4675	1.402	2.485	3.615
15dB	1.010	0	0.467	1.402	2.485
20dB	6.697	1.010	0	0.467	1.402
25dB	27.174	6.699	1.011	0	0.467
30dB	94.394	27.168	6.697	1.010	0

C. Unknown SNR

We have shown that an attempt to estimate d when the amplitude is fluctuating according to a Rayleigh distribution would require a large number of measurements. To circumvent this issue, we adopt an alternative approach where we do not attempt to estimate d. Instead we marginalise out the parameter d over the range of possible values and find a likelihood for g_a which is not conditional on d.

Suppose that $p(d)$, defined on the support of possible SNR values $[d_1, d_2]$ gives the expected probability distribution of SNR values. Then we can define the likelihood and probability of detection as

$$g_a(a) := \int_{d_1}^{d_2} p(\delta) g_a(a|\delta) d\delta \tag{36}$$

$$p_D^{\tau} := \int_{d_1}^{d_2} p(\delta) p_D^{\tau}(\delta) d\delta \tag{37}$$

Whilst the obvious choice for our distribution on d may be uniform, this in fact biases higher dB targets since we have a logarithmic scale. Also, as d increases, there is little difference between the different distributions, so we have chosen to bias $p(d)$ in favour of a distribution which has a more uniform spread in the dB scale for targets.

We can find an uninformative prior that is not dependent on the parameter space using the Fisher Information. The Jeffreys prior [15] in Bayesian statistics is an uninformative (objective) prior distribution on parameter space that is proportional to the square root of the Fisher information,

$$p(d) \propto \sqrt{I(d)}. \tag{38}$$

The fact that this is not dependent on the parameter variable means that this is appropriate for any change in this variable. For our case, we consider the Fisher Information for the Rayleigh single-object amplitude likelihood with one measurement in equation (31), which gives

$$p(d) \propto \frac{1}{(1+d)}, \tag{39}$$

where $p(d)$ is suitably normalised for the region $[d_1, d_2]$.

This prior gives a uniform distribution in the dB domain, which can be shown as follows: Suppose that we have a uniform distribution $u(\xi)$ in the dB domain. Then, if we wish to integrate this over a range of dB values $[dB_1, dB_2]$ and

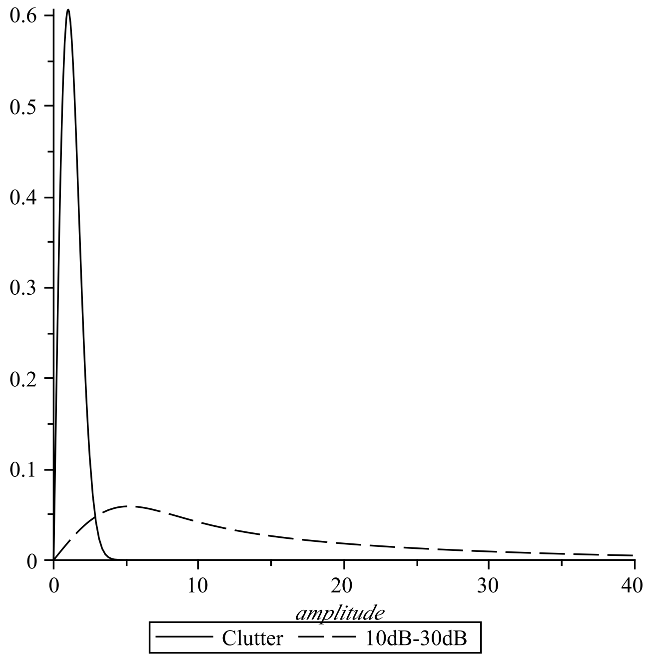

Figure 2. Amplitude pdfs for Rayleigh likelihoods: clutter and unknown SNR.

convert this into d domain, we can change the variable of integration as follows

$$\int_{dB_1}^{dB_2} u(\xi) d\xi = \int_{\phi^{-1}(dB_1)}^{\phi^{-1}(dB_2)} u(\phi(\delta))\phi'(\delta) d\delta \propto \int_{\phi^{-1}(dB_1)}^{\phi^{-1}(dB_2)} \phi'(\delta) d\delta, \tag{40}$$

where $\phi(d) = 10\log_{10}(1+d)$ is the function that converts d into dB. Hence $p(d) = \phi'(d) \propto \frac{1}{1+d}$ gives us the required function in the d domain.

A significant advantage of using distribution (39) is that our likelihood function g_a in equation (36), which has been marginalised over d now has an analytic solution,

$$g_a(a) = \frac{2\left(\exp\left(\frac{-a^2}{2(1+d_2)}\right) - \exp\left(\frac{-a^2}{2(1+d_1)}\right)\right)}{a(\ln(1+d_2) - \ln(1+d_1))}. \tag{41}$$

When the target SNR is unknown we apply the theory presented in Section III with only difference that the target amplitude likelihood of (8) simplifies to $g_a(a)$ given by (41). This means that numerical integration is not necessary and hence the computation required is significantly lower. The probability of target detection in equation (8) over the marginalised region $[d_1, d_2]$ can be computed with numerical integration offline since it does not need to be computed for each iteration. Figure 2 shows the amplitude likelihoods for false alarm and the uknown likelihood for a target from equation (41) where d_1 and d_2 correspond to $10dB$ and $30dB$ respectively.

We can analyse the quality of this approximation for given target SNRs in the region $[d_1, d_2]$ by computing the Kullback-Leibler divergence between targets with fixed SNR and the marginalised likelihood

$$D(g_a(a|d)||g_a(a)) := \int_{a=0}^{\infty} g_a(a|d) \ln\left(\frac{g_a(a|d)}{g_a(a)}\right) da. \tag{42}$$

Table III shows the Kullback-Leibler divergence for targets in the region $10dB - 30dB$ for a range of assumed fixed SNR values. All of the values shown here are less than $1nat$, hence the marginalised likelihood more accurately characterises the target SNR than any particular fixed choice in table II.

In practice, the target SNR is not fixed but is in fact dependent on aspect angle and hence will fluctuate. Our approach is more robust under these circumstances since it works for a range of different SNR and does not assume that the target SNR is fixed.

Table III
KULLBACK-LEIBLER DIVERGENCE BETWEEN TRUE AND MARGINALISED AMPLITUDE DISTRIBUTIONS (IN NATS)

Marginalised	10 dB - 30 dB
True	
10dB	0.966
15dB	0.411
20dB	0.204
25dB	0.317
30dB	0.966

V. SIMULATIONS

In this section, we have implemented the proposed pseudo-likelihoods for the Gaussian mixture PHD and CPHD filters [8], [16], [9], although this approach could be easily incorporated into particle implementations of these filters [17], or other algorithms derived from FISST [18]. We examine the performance of the proposed techniques by considering the errors for known and unknown SNR with different probabilities of false alarm p^{τ}_{FA} and for different SNR values.

To examine the performance of these techniques, we benchmark them against the corresponding PHD/CPHD filters which do not use the amplitude information. In order to do so, we require a proper notion of the error between the estimated and true multi-target states, which is indeed fundamental for the performance evaluation of any filter. As argued in [19] however, traditional measures of performance can exhibit elements of arbitrariness and hence may not be entirely suitable for performance evaluation. Consequently, to ensure a fair and balanced assessment, a formal notion of a multi-target miss-distance is required; that is, a formal metric is required. While conventional metrics such as the Haussdorff of Wasserstein metrics may be obvious candidates, it is demonstrated in [19] that these do not admit physically consistent interpretations and thus are not entirely suitable for performance evaluation. Accordingly, we adopt the Optimal Sub-Pattern Assignment (OSPA) metric or multi-target miss-distance proposed in [19] for the purpose of multi-target performance evaluation. We use the OSPA metric because it jointly captures differences in the cardinality and the individual elements between two finite sets in a mathematically consistent yet intuitively meaningful way [19].

A. OSPA metric

An intuitive construction of the OSPA metric is first presented. Suppose that we are given two finite sets each with arbitrary cardinality (number of elements), e.g. the true and estimated multi-target state sets. The set with the smaller cardinality is initially chosen as a reference. The optimum assignment of each point in the reference set to exactly and exclusively one point in the other set is then calculated, where optimum means a minimum total distance, subject to the constraint that distances are capped at a preselected maximum or cut-off value. The optimum assignment and resulting summed errors can be interpreted as the "localization errors", which are determined by giving each point the "benefit of the doubt". All other points which remain unassigned are also charged with an error value, where each extraneous point is penalized at the maximum or cut-off value. These errors can interpreted as "cardinality errors" which are "penalized at the maximum rate". The "total error" committed is then the sum of the "total localization errors" and the "total cardinality errors". Remarkably, if the "total error" is divided by the larger cardinality of the two given sets, the resulting "per target error" is proper distance in the formal metric sense.

A formal statement of the OSPA metric is now shown. The OSPA metric $\bar{d}_p^{(c)}$ is defined as follows. Let $d^{(c)}(x,y) := \min(c, \|x-y\|)$ for $x,y \in \mathcal{X}$, and Π_k denote the set of permutations on $\{1,2,\ldots,k\}$ for any positive integer k. Then, for $p \geq 1$, $c > 0$, and $X = \{x_1,\ldots,x_m\}$ and $Y = \{y_1,\ldots,y_n\}$,

$$\bar{d}_p^{(c)}(X,Y) := \left(\frac{1}{n}\left(\min_{\pi\in\Pi_n}\sum_{i=1}^{m} d^{(c)}(x_i,y_{\pi(i)})^p + c^p(n-m)\right)\right)^{\frac{1}{p}} \tag{43}$$

if $m \leq n$, and $\bar{d}_p^{(c)}(X,Y) := \bar{d}_p^{(c)}(Y,X)$ if $m > n$; and $\bar{d}_p^{(c)}(X,Y) = \bar{d}_p^{(c)}(Y,X) = 0$ if $m = n = 0$. The OSPA distance is interpreted as a p-th order per-target error, comprised of a p-th order per-target localization error and a p-th order per-target cardinality error. These components are defined as

$$\begin{aligned} \bar{e}_{loc}^{(c)}(X,Y) &:= \left(\frac{1}{n}\cdot\min_{\pi\in\Pi_n}\sum_{i=1}^{m} d^{(c)}(x_i,y_{\pi(i)})^p\right)^{1/p}, \\ \bar{e}_{card}^{(c)}(X,Y) &:= \left(\frac{c^p(n-m)}{n}\right)^{1/p} \end{aligned} \tag{44}$$

if $m \leq n$, and $\bar{e}_{loc}^{(c)}(X,Y) := \bar{e}_{loc}^{(c)}(Y,X)$, $\bar{e}_{card}^{(c)} := \bar{e}_{card}^{(c)}(Y,X)$ if $m > n$. The order parameter p determines the sensitivity of the metric to outliers, and the cut-off parameter c determines the relative weighting of the penalties assigned to cardinality and localization errors. When $p = 1$, the OSPA distance can be interpreted exactly as the "per-target error" comprising the sum of the "per-target localization error" and the "per-target cardinality error". The OSPA metric is also easily computable, for details see [19].

B. Numerical results

Up to $N(k) = 10$ targets are generated with 4 dimensional state vectors containing the 2 dimensional positions and velocities of targets following a constant velocity model. All of the targets in each simulation had the same mean SNR (this is not required by the algorithm, but simplifies the presentation of results). The probability of survival for each target was $p_S = 0.9$, which was used both in the target generation and in the PHD/CPHD filter predictions. The measurements contain the positions and amplitudes of the measurements with Gaussian noise. The position data is in the surveillance region $[0\ 1000;\ 0\ 1000]$ with 1024 measurements before thresholding[2] according to τ. The Gaussian process noise variance is 1.0, and the measurement noise variance is 5.0. The birth intensity is 0.05 over 4 birth Gaussians in the surveillance region. To make the assessment as fair as possible, the probability of detection p_D^τ for the filters without amplitude information was chosen to be the same as that of the unknown SNR case, as in Table I. Note that the chosen value of OSPA metric c $(100m)$ is significantly larger than the typical measurement noise, but significantly smaller than the maximal distance between objects, and thus maintains a balance between the cardinality and localization components of the OSPA error. The amplitude likelihood for the unknown SNR is taken to be the whole $10dB$ to $30dB$ spectrum of expected target SNR values.

Tables II and III give the mean and standard deviation of the errors for 100 Monte Carlo runs with the PHD and CPHD filters with $p = 1$, and $c = 100m$. For the PHD and CPHD filters, we have presented tables for the combined OSPA metric, the localisation estimate, and the cardinality estimate. Tables II and III are presented in three block of three: The first third of the table, (1a)-(1c), shows results for the combined metric, the second third, (2a)-(2c), show the localisation results, and the third, (3a)-(3c), shows the results for the cardinality estimates. Each block has results for the known SNR likelihood, (1a), (2a), (3a), the unknown SNR likelihood, (1b), (2b), (3b), and the filter without amplitude information, (1c), (2c), (3c).

1) PHD filter: Figures 3, 4 and 5 show an example of the PHD filter tracking of 30dB targets with $p_{FA}^\tau = 0.1$ (which means that we have an expected number of false alarms of around 100 per scan) along with the mean cardinality error. The crosses are the measurements, circles the true positions and lines representing the tracked trajectories. This is the case where the method using the amplitude information works best both for unknown and known SNR, as the clutter distribution will poorly represent the target amplitude, and there is a big distinction in the target and clutter distributions. In this case, having more measurements aids the method using the amplitude, since we discard less useful information, while at the same time this harms the filter without amplitude since there are many more false alarms. The PHD filter without amplitude in this example did not identify any targets (so the mean cardinality error is equal to the mean number of targets), which can be seen by the fact that there are no tracks in the figure. Using the amplitude information, there was a mean cardinality error of 0.3664 targets per scan for the unknown SNR and 0.0563 for the known SNR.

[2]Note that in our simulations we do not have a finite sensor resolution: the measurements can be generated from anywhere in the surveillance region.

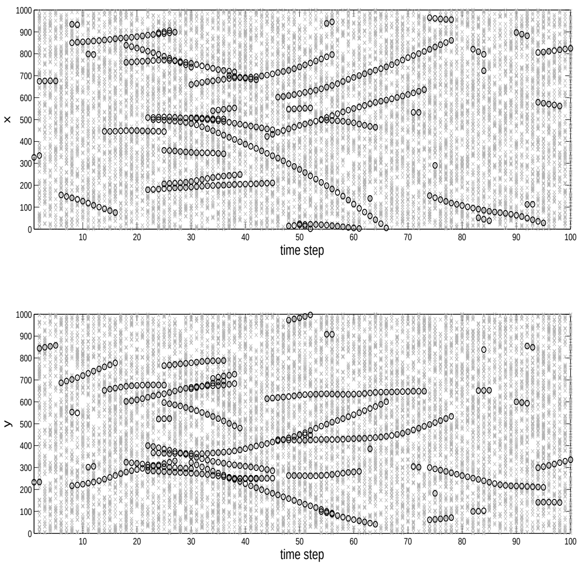

Figure 3. Without amplitude - mean cardinality error = 4.0526

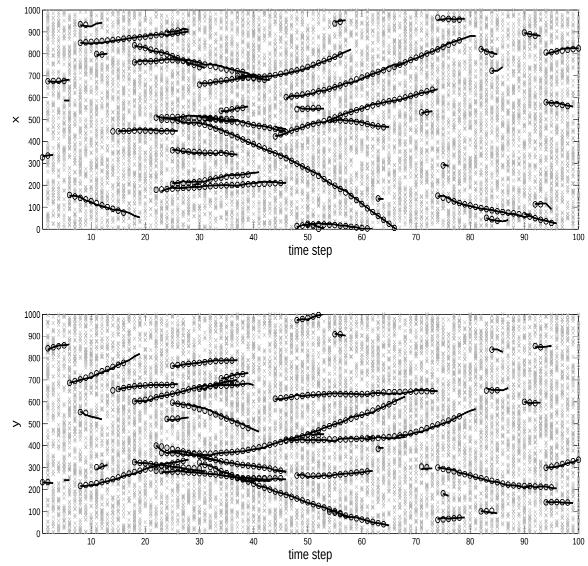

Figure 5. Unknown SNR - mean cardinality error = 0.3664

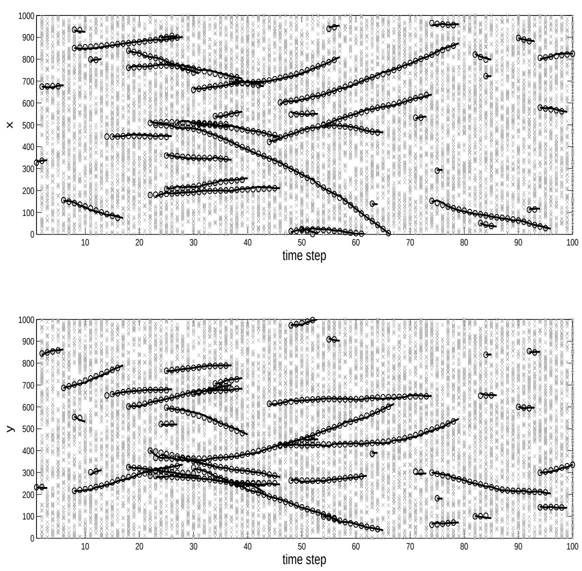

Figure 4. Known SNR - mean cardinality error = 0.0563

Looking at the combined OSPA metric in subtables (1a)-(1c), we see that in all cases the PHD filter using amplitude likelihoods for known and unknown SNR give better performance than the PHD filter without using amplitude information. For targets with SNR in the range 10dB to 15dB, as p_{FA} increases the PHD filters with amplitude likelihoods have improved performance whereas without there is a deterioration in performance. This can be seen for $p_{FA} = 0.1$, where the PHD filter without amplitude information has very high values in subtable (1c), with corresponding cardinality estimate in subtable (3c). One possible explanation for this is that fewer true measurements are being thresholded out and the amplitude helps discriminate better whereas the PHD filter without this information may have more difficulty discriminating with more false alarms.

Comparing the tables (1a) and (1b), we see for high SNR the PHD filter with known amplitude likelihood is consistent as the number of false alarms increases whereas the PHD filter with unknown amplitude likelihood sees a slight deterioration. Looking at tables for the localization, (2a) and (2b), and cardinality, (3a) and (3b), we see that this comes from the cardinality estimate, the PHD filter with known amplitude remains around the same value as p_{FA} increases whereas for unknown amplitude the errors increase. The localisation errors in these cases are similar.

2) CPHD filter: Similar conclusions can be drawn with the CPHD filter as above, although there are some differences with the PHD filter. For known amplitude, subtables (1a), the CPHD filter and PHD filter are comparable for SNR for 25dB to 30dB, but for unknown amplitude, subtables (1b), perhaps surprisingly, the PHD filter does better. Looking at the cardinality estimates in subtables (3b), we see that the CPHD filter gives worse performance for a low number of false alarms but improves as p_{FA} increases, compared to the PHD filter having good estimates for low number of false alarms but the error increases slightly as false alarms increase. One possible explanation is that the CPHD filter is less responsive to changes in target number and hence the amplitude likelhood has less effect here, whereas the PHD filter is more responsive but also has more false estimates in cardinality in general.

VI. CONCLUSIONS

In this paper we show how to incorporate detection amplitude information into the generic multi-object Bayes filter and its computationally feasible approximations, the PHD filter and the CPHD filter. We also investigate the amplitude feature likelihoods used in conventional target tracking and find that one of the limitations of this approach is that prior knowledge of target SNR is needed for the model to work effectively. Using the Fisher Information, we show that estimating this property requires a significant number of measurements from the target. Instead of trying to estimate the SNR of the target, we propose an alternative approach where the SNR is marginalised over a range of possible values, which results in an analytic solution for Rayleigh target likelihoods. Based on the assumption that the amplitudes of the measurements from true targets are stronger than those from clutter, i.e. the signal to noise ratio is within a given range, we found this can significantly improve the performance of the filter. Furthermore, there is little increase in the computational complexity using this approach.

Using the Gaussian mixture implementations of PHD and CPHD filters, we conclude from simulations that using amplitude feature information results in a more stable and accurate estimate of the number of targets. In high SNR scenarios, the PHD filter even outperformed the CPHD filter despite having a much lower complexity, by efficiently exploiting the amplitude likelihood. In low SNR scenarios however, the CPHD filter generally outperformed the PHD filter, as the former was able to tolerate harsher conditions with the inclusion of the amplitude likelihood. Future work will consider an application of amplitude-enhanced PHD and CPHD filters to real sonar data.

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Table IV
PHD RESULTS FOR OSPA METRIC (MEAN ERROR/ STANDARD DEVIATION): A - KNOWN SNR; B - UNKNOWN SNR; C - NO AMPLITUDE INFORMATION; 1 - OVERALL OSPA DISTANCE; 2 - OSPA LOCALISATION DISTANCE; 3 - OSPA CARDINALITY DISTANCE.

p_{FA}	SNR τ	10 dB	15 dB	20 dB	25dB	30 dB
(1a)						
0.0010	3.7169	65.854 / 3.744	24.596 / 2.791	13.422 / 1.874	8.190 / 1.007	6.722 / 0.371
0.0100	3.0349	49.157 / 2.398	21.730 / 2.287	12.616 / 1.777	8.142 / 1.068	6.622 / 0.661
0.0500	2.4477	41.855 / 3.804	19.676 / 1.678	11.673 / 1.768	8.009 / 0.571	6.708 / 0.553
0.1000	2.1460	39.148 / 4.276	18.760 / 2.088	11.237 / 1.393	7.995 / 1.265	6.687 / 0.633
(1b)						
0.0010	3.7169	57.176 / 4.015	24.890 / 2.814	13.260 / 1.798	8.216 / 1.009	6.867 / 0.448
0.0100	3.0349	50.377 / 3.888	22.319 / 2.421	12.573 / 1.731	8.361 / 0.902	7.066 / 0.533
0.0500	2.4477	49.074 / 4.652	20.642 / 2.391	11.525 / 1.645	8.787 / 0.859	7.628 / 0.792
0.1000	2.1460	48.721 / 3.534	20.191 / 2.393	11.028 / 1.240	8.428 / 1.228	7.671 / 0.625
(1c)						
0.0010	3.7169	64.686 / 4.780	33.930 / 3.078	23.201 / 1.956	18.600 / 1.768	17.417 / 1.499
0.0100	3.0349	66.830 / 5.673	45.500 / 5.109	36.569 / 3.220	32.242 / 3.786	31.980 / 4.390
0.0500	2.4477	82.039 / 5.501	71.439 / 6.290	66.196 / 4.942	64.946 / 8.272	65.641 / 6.673
0.1000	2.1460	95.550 / 4.774	96.700 / 3.854	96.850 / 3.438	96.000 / 4.942	97.250 / 4.351
(2a)						
0.0010	3.7169	38.288 / 5.731	5.028 / 0.494	5.636 / 0.389	5.826 / 0.385	6.023 / 0.301
0.0100	3.0349	21.269 / 2.507	5.277 / 0.490	5.812 / 0.385	5.660 / 0.344	5.888 / 0.239
0.0500	2.4477	8.058 / 2.347	5.822 / 0.644	5.695 / 0.380	5.936 / 0.330	5.877 / 0.318
0.1000	2.1460	6.919 / 1.327	5.975 / 0.692	5.791 / 0.292	5.640 / 0.406	5.848 / 0.365
(2b)						
0.0010	3.0349	3.283 / 0.304	4.896 / 0.437	5.659 / 0.385	5.866 / 0.398	6.058 / 0.323
0.0100	3.7169	3.772 / 0.557	5.089 / 0.329	5.826 / 0.372	5.716 / 0.349	5.894 / 0.237
0.0500	2.4477	3.743 / 0.568	5.519 / 0.587	5.790 / 0.353	5.965 / 0.357	5.911 / 0.345
0.1000	2.1460	4.045 / 0.676	5.583 / 0.598	5.845 / 0.290	5.653 / 0.411	5.838 / 0.404
(2c)						
0.0010	3.7169	2.671 / 0.588	4.076 / 0.532	4.745 / 0.509	4.991 / 0.431	5.091 / 0.299
0.0100	3.0349	3.854 / 0.895	4.803 / 0.891	5.717 / 0.634	5.465 / 0.925	5.794 / 0.455
0.0500	2.4477	9.071 / 2.595	10.010 / 2.180	10.307 / 2.191	10.290 / 2.181	10.843 / 2.363
0.1000	2.1460	0.000 / 0.000	0.000 / 0.000	0.000 / 0.000	0.000 / 0.000	0.000 / 0.000
(3a)						
0.0010	3.7169	27.566 / 3.365	19.568 / 2.794	7.786 / 1.958	2.365 / 0.968	0.700 / 0.326
0.0100	3.0349	27.888 / 2.176	16.453 / 2.330	6.804 / 1.855	2.481 / 1.082	0.734 / 0.603
0.0500	2.4477	33.797 / 4.582	13.853 / 1.691	5.978 / 1.962	2.073 / 0.582	0.831 / 0.596
0.1000	2.1460	32.229 / 4.216	12.785 / 2.129	5.446 / 1.413	2.355 / 1.270	0.839 / 0.490
(3b)						
0.0010	3.7169	53.893 / 4.082	19.994 / 2.814	7.601 / 1.873	2.350 / 0.979	0.808 / 0.487
0.0100	3.0349	46.606 / 3.984	17.230 / 2.342	6.748 / 1.767	2.644 / 0.986	1.172 / 0.468
0.0500	2.4477	45.331 / 4.764	15.123 / 2.328	5.735 / 1.757	2.823 / 0.858	1.717 / 0.795
0.1000	2.1460	44.676 / 3.581	14.608 / 2.463	5.184 / 1.265	2.775 / 1.194	1.833 / 0.628
(3c)						
0.0010	3.7169	62.015 / 4.910	29.854 / 3.038	18.456 / 2.161	13.610 / 1.746	12.326 / 1.498
0.0100	3.0349	62.976 / 6.017	40.697 / 5.578	30.852 / 3.439	26.776 / 4.071	26.186 / 4.500
0.0500	2.4477	72.968 / 6.030	61.429 / 6.022	55.889 / 5.366	54.656 / 8.099	54.798 / 7.065
0.1000	2.1460	95.550 / 4.774	96.700 / 3.854	96.850 / 3.438	96.000 / 4.942	97.250 / 4.351

Table V

CPHD RESULTS FOR OSPA METRIC (MEAN ERROR/ STANDARD DEVIATION): A - KNOWN SNR; B - UNKNOWN SNR; C - NO AMPLITUDE INFORMATION; 1 - OVERALL OSPA DISTANCE; 2 - OSPA LOCALISATION DISTANCE; 3 - OSPA CARDINALITY DISTANCE.

p_{FA}	SNR / τ	10 dB	15 dB	20 dB	25dB	30 dB
(1a)						
0.0010	3.7169	51.641 / 3.548	23.172 / 2.481	13.770 / 1.540	8.135 / 1.000	6.659 / 0.369
0.0100	3.0349	42.796 / 3.540	21.763 / 2.652	12.465 / 1.922	8.016 / 0.920	6.512 / 0.515
0.0500	2.4477	41.831 / 3.717	20.542 / 1.473	11.350 / 1.554	8.052 / 0.933	6.625 / 0.598
0.1000	2.1460	40.398 / 4.966	18.820 / 1.799	10.633 / 1.228	7.631 / 1.313	6.611 / 0.613
(1b)						
0.0010	3.7169	50.974 / 3.656	23.382 / 2.507	15.258 / 1.625	12.813 / 1.227	12.570 / 1.239
0.0100	3.0349	46.887 / 4.180	22.037 / 1.985	13.201 / 1.761	10.141 / 1.260	9.154 / 0.651
0.0500	2.4477	45.737 / 4.910	19.671 / 2.123	11.675 / 1.485	9.716 / 0.856	9.169 / 1.137
0.1000	2.1460	44.615 / 3.509	18.550 / 2.133	11.322 / 1.260	9.121 / 1.466	8.686 / 0.909
(1c)						
0.0010	3.7169	62.933 / 4.705	34.094 / 3.419	25.405 / 2.896	21.669 / 2.606	21.055 / 2.389
0.0100	3.0349	66.332 / 4.838	49.477 / 5.767	39.382 / 4.111	37.832 / 4.401	36.137 / 4.084
0.0500	2.4477	76.572 / 4.401	67.213 / 6.494	58.505 / 5.771	59.200 / 4.886	60.205 / 5.400
0.1000	2.1460	95.550 / 4.774	96.605 / 3.833	96.443 / 3.349	95.976 / 4.923	96.575 / 4.317
(2a)						
0.0010	3.7169	24.901 / 2.838	8.352 / 1.364	6.538 / 0.621	5.824 / 0.382	6.043 / 0.294
0.0100	3.0349	18.678 / 2.009	8.075 / 0.842	5.909 / 0.465	5.669 / 0.349	5.902 / 0.250
0.0500	2.4477	16.737 / 2.122	8.272 / 1.163	6.020 / 0.547	5.978 / 0.316	5.960 / 0.330
0.1000	2.1460	16.759 / 2.894	7.994 / 0.967	5.977 / 0.333	5.664 / 0.394	5.874 / 0.366
(2b)						
0.0010	3.0349	6.205 / 1.142	7.724 / 1.352	7.302 / 0.843	6.194 / 0.504	5.884 / 0.306
0.0100	3.7169	5.188 / 1.068	6.698 / 0.987	6.448 / 0.733	5.935 / 0.481	5.866 / 0.295
0.0500	2.4477	4.913 / 0.841	6.557 / 0.810	6.322 / 0.584	6.052 / 0.398	5.887 / 0.361
0.1000	2.1460	5.649 / 0.980	6.405 / 0.801	6.168 / 0.357	5.739 / 0.469	5.930 / 0.474
(2c)						
0.0010	3.7169	7.970 / 2.242	8.495 / 1.415	7.457 / 1.379	6.518 / 0.958	6.336 / 0.672
0.0100	3.0349	12.532 / 2.076	12.355 / 2.500	10.384 / 1.352	9.765 / 1.644	10.444 / 1.992
0.0500	2.4477	23.359 / 4.002	20.398 / 4.408	17.164 / 3.407	20.138 / 4.441	18.955 / 3.941
0.1000	2.1460	0.000 / 0.000	0.005 / 0.018	0.018 / 0.048	0.001 / 0.006	0.069 / 0.158
(3a)						
0.0010	3.7169	26.740 / 4.023	14.819 / 2.457	7.233 / 1.561	2.310 / 0.960	0.616 / 0.312
0.0100	3.0349	24.118 / 3.531	13.688 / 2.770	6.556 / 2.043	2.348 / 0.985	0.610 / 0.432
0.0500	2.4477	25.094 / 4.068	12.271 / 1.384	5.330 / 1.669	2.074 / 0.897	0.665 / 0.501
0.1000	2.1460	23.639 / 3.725	10.825 / 2.069	4.657 / 1.224	1.967 / 1.300	0.736 / 0.508
(3b)						
0.0010	3.7169	44.768 / 3.823	15.658 / 2.470	7.956 / 1.446	6.618 / 1.199	6.686 / 1.206
0.0100	3.0349	41.699 / 4.782	15.339 / 2.386	6.753 / 1.849	4.206 / 1.235	3.288 / 0.789
0.0500	2.4477	40.824 / 4.802	13.114 / 2.058	5.352 / 1.465	3.665 / 0.979	3.281 / 1.172
0.1000	2.1460	38.966 / 3.376	12.145 / 2.096	5.154 / 1.314	3.383 / 1.361	2.756 / 0.833
(3c)						
0.0010	3.7169	54.963 / 4.975	25.600 / 3.398	17.948 / 3.203	15.151 / 2.685	14.719 / 2.445
0.0100	3.0349	53.800 / 4.983	37.122 / 6.039	28.998 / 3.720	28.067 / 4.088	25.694 / 3.524
0.0500	2.4477	53.213 / 5.366	46.815 / 5.668	41.342 / 6.495	39.062 / 4.859	41.250 / 6.480
0.1000	2.1460	95.550 / 4.774	96.600 / 3.832	96.425 / 3.352	95.975 / 4.922	96.506 / 4.359