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Robust Multi-Bernoulli Filtering

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Abstract

In Bayesian multi-target filtering knowledge of parameters such as clutter intensity and detection probability profile are of critical importance. Significant mismatches in clutter and detection model parameters results in biased estimates. In this paper we propose a multi-target filtering solution that can accommodate non-linear target models and an unknown non-homogeneous clutter and detection profile. Our solution is based on the multi-target multi-Bernoulli filter that adaptively learns non-homogeneous clutter intensity and detection probability while filtering.

Index Terms

Multi-Target Bayes filter, multi-Bernoulli filter, multi-target tracking, online parameter estimation, robust filtering, finite set statistics.

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I. INTRODUCTION

Multi-target filtering is a challenging problem which involves the joint detection and estimation of a time varying and unknown number of targets based on incoming data. Measurements produced by an imperfect sensor are usually subjected to noise, missed detections and false alarms. To cope with non-ideal sensors, the majority of multi-target tracking filters assume a sensor model, which describes the statistical behaviour of the collected returns [1], [2], [16]. The estimation performance of these filters is generally tolerant to mismatches in the noise model, since the setting of a reasonably large noise variance can reasonably accommodate most situations. However, mismatches in the specification of the detection and clutter model parameters can lead to a significant bias or even complete corruption of filter estimates.

The random finite set (RFS) paradigm [16] is a mathematically principled and elegant approach to multi-target filtering which has attracted considerable attention in recent times. In particular, the development of the PHD and CPHD filters [14], [15], along with implementations [25]–[27], and convergence results [3], [5], [6], [11], [25], has demonstrated the utility of the RFS approach. However, the majority of RFS based approaches so far have assumed a priori specification of the detection and clutter parameters, except for the approach in [24] which attempts to estimate the entire suite of multi-target model parameters.

Recent works addressing the filtering problem with unknown clutter and detection parameters are based on the CPHD and PHD filters [17]–[19]. The work in [19] further proposes closed form implementations via Beta-Gaussian mixtures which are only applicable to linear Gaussian target dynamics. Mild non-linearities are however briefly addressed in [19] via extended and unscented Kalman techniques. A maximum likelihood approach to clutter intensity estimation combined with PHD filtering was also proposed in [4]. Although it is possible to extend these works to general non-linear scenarios via particle implementations, such a solution may not yield acceptable performance, since the necessary end process of performing clustering to yield state estimates is inherently unreliable. Furthermore, it would be difficult to diagnose the cause of any poor performance, since it could be the result of the clustering process or the filtering itself, or both.

Although the PHD and CPHD filters are synonymous with the RFS approach, another important but lesser known class of filters from this framework are the (multi-target) multi-Bernoulli filters [16], [28], [30]. A number of implementations and extensions have also been considered in [36], [37], [38], [20], [21]. Unlike the PHD and CPHD recursions, which propagate moments and cardinality distributions, the multi-Bernoulli filter propagates the parameters of a multi-Bernoulli distribution that approximate the posterior multi-target density. The multi-Bernoulli filter has the same complexity as the PHD filter, i.e. linear in the number of targets and linear in the number measurements, and the performance of the multi-target multi-Bernoulli filter is expected to be similar to PHD filter. However, with the particle or Sequential Monte Carlo (SMC) implementation, the multi-Bernoulli filter is advantageous in that it does not require the additional clustering step for multi-target state estimation. Formal convergence results for the particle implementation were established in [13].

The multi-Bernoulli filter has been applied to tracking in sensor networks [31], [32], [33], [34], [39], [12] and tracking from audio and video data [8]. The first RFS based filter for image data was proposed in [30] using multi-Bernoulli approximation, which has further been applied to visual tracking and cell tracking [7], [9], [10]. Hybrid multi-Bernoulli and Poisson multi-target filters were also considered in [35].

To date, all of the works on the multi-Bernoulli filter have assumed a priori knowledge of the clutter intensity and detection profile, with the exception of the multi-Bernoulli filter for image measurements [30] which avoids detections and clutter altogether. This paper addresses the problem of multi-target filtering with unknown clutter intensity and detection profile for non-linear problems using the multi-Bernoulli filter. The main advantage of the multi-Bernoulli approach is the capability to accommodate non-linear models, because its particle implementations do not require clustering. In addition, the proposed approach is naturally able to accommodate an unknown and non-homogenous clutter intensity and detection profile. The multi-Bernoulli filter is thus a natural candidate to develop into a solution which is applicable to general non-linear models.

Although the multi-Bernoulli and CPHD/PHD filters are all approximations to the multi-target Bayes filter, the fundamental difference between them is the following: the former is a parameterized density approximation while the latter are moment based approximations. The proposed robust multi-Bernoulli filtering solution is fundamentally different to the formulation in [19], and addresses problems which are not possible with the approach in [19]. Preliminary results have been reported in the conference publication [29]. Analytic implementations along the lines of the Beta-Gaussian solution proposed in [19] are also possible, however this offers no obvious advantages over the CPHD and PHD filters in [19].

The paper is organized as follows. Section II reviews the standard multi-Bernoulli filter. Section III presents an adaption of the multi-Bernoulli filter to accommodate a time varying, unknown and non-homogenous clutter intensity and detection probability. Section IV then presents a particle implementation. Section V shows numerical studies with a non-linear multi-target filtering scenario. Concluding remarks are given in Section VI.

II. THE MULTI-BERNOULLI FILTER

Suppose that at time k, there are $N(k)$ target states $\breve{x}_{k,1}, \ldots, \breve{x}_{k,N(k)}$, each taking values in a state space $\breve{\mathcal{X}}$, and $M(k)$ observations $z_{k,1}, \ldots, z_{k,M(k)}$ each taking values in an observation space $\mathcal{Z}$. Then, the multi-target state and multi-target observations, at time k, are the finite sets [14], [16], [28]

$$\breve{X}_k = \{\breve{x}_{k,1}, \ldots, \breve{x}_{k,N(k)}\} \subset \breve{\mathcal{X}},$$

$$Z_k = \{z_{k,1}, \ldots, z_{k,M(k)}\} \subset \mathcal{Z}.$$

The multi-target Bayes filter is the fundamental mechanism for propagating the multi-target posterior density recursively in time. The stochastic variability of the multi-target state and multi-target observation are captured by modelling with RFSs. In essence, an RFS is simply a finite-set-valued random variable.

In this paper, we focus on the parameterized classes of Bernoulli and multi-Bernoulli RFSs [16], [28]. A *Bernoulli* RFS $\breve{X}$ on $\breve{\mathcal{X}}$ has probability $1-r$ of being empty, and probability r of being a singleton whose only element is distributed according to a probability density p defined on $\breve{\mathcal{X}}$. The mean cardinality of a Bernoulli RFS is r. A Bernoulli RFS is completely described by the parameter pair (r, p) and its probability density is given by

$$\pi(\breve{X}) = \begin{cases} 1-r & \breve{X} = \emptyset \\ r \cdot p(\breve{x}) & \breve{X} = \{\breve{x}\} \\ 0 & \text{otherwise} \end{cases}, \tag{1}$$

A *multi-Bernoulli* RFS $\breve{X}$ on $\breve{\mathcal{X}}$ is a union of a fixed number of independent Bernoulli RFSs $\breve{X}^{(i)}$ with existence probability $r^{(i)} \in (0,1)$ and probability density $p^{(i)}$ defined on $\breve{\mathcal{X}}$ for $i = 1, ..., M$, i.e. $X = \bigcup_{i=1}^{M} X^{(i)}$.The mean cardinality of a multi-Bernoulli RFS is $\sum_{i=1}^{M} r^{(i)}$. A multi-Bernoulli RFS is completely described by the parameter set $\{(r^{(i)}, p^{(i)})\}_{i=1}^{M}$, see [16], [28] for further details.

The (cardinality-balanced) multi-Bernoulli filter [28] propagates the posterior density of the multi-target state, as a multi-Bernoulli density recursively in time, given a sequence of multi-target observations. The multi-Bernoulli filter is consequently a parameterized approximation to the multi-target Bayes filter [28]. The filter has the intuitive interpretation of propagating a time varying number of target tracks, each of which is described by a probability of existence as well as a probability density on the state variable.

The prediction and update steps of the multi-Bernoulli filter [28] are summarized directly below. The following notation is used for the specification of the standard single target model which describes the transition, survival/death, and birth process of individual targets, as well as the likelihood, detection/missed-detection and Poisson clutter or false alarm process:

$$p_{S,k}(\breve{\zeta}) = \textit{probability of survival to time } k\textit{, given previous state } \breve{\zeta},$$

$$p_{D,k}(\breve{x}) = \textit{probability of detection at time } k\textit{, for current state } \breve{x},$$

$$f_{k|k-1}(\cdot|\breve{\zeta}) = \textit{single target transition to time } k\textit{,given previous state } \breve{\zeta},$$

$$g_k(\cdot|\breve{x}) = \textit{single target likelihood at time } k\textit{, given current state } \breve{x},$$

$$\{(r_{\Gamma,k}^{(i)}, p_{\Gamma,k}^{(i)})\}_{i=1}^{M_{\Gamma,k}} = \textit{parameters of the multi-Bernoulli RFS of births at time } k,$$

$$\kappa_k(\cdot) = \textit{intensity function of Poisson clutter at time } k,$$

$$\langle \alpha, \beta \rangle = \int \alpha(x)\beta(x)dx \text{ (the inner product between real functions } \alpha \text{ and } \beta)$$

$$\langle \alpha, \beta \rangle = \sum_{\ell=0}^{\infty} \alpha(\ell)\beta(\ell) \text{ (the inner product between real sequences } \alpha \text{ and } \beta)$$

Standard Prediction: If at time $k-1$, the posterior multi-target density is multi-Bernoulli of the form

$$\pi_{k-1} = \{(r_{k-1}^{(i)}, p_{k-1}^{(i)})\}_{i=1}^{M_{k-1}},$$

then the predicted multi-target density is multi-Bernoulli given by a union of new birth and persisting or predicted components

$$\pi_{k|k-1} = \{(r_{\Gamma,k}^{(i)}, p_{\Gamma,k}^{(i)})\}_{i=1}^{M_{\Gamma,k}} \cup \{(r_{P,k|k-1}^{(i)}, p_{P,k|k-1}^{(i)})\}_{i=1}^{M_{k-1}}, \tag{2}$$

where

$$r_{P,k|k-1}^{(i)} = r_{k-1}^{(i)} \langle p_{k-1}^{(i)}, p_{S,k} \rangle, \tag{3}$$

$$p_{P,k|k-1}^{(i)}(\breve{x}) = \frac{\langle f_{k|k-1}(\breve{x}|\cdot), p_{k-1}^{(i)} p_{S,k} \rangle}{\langle p_{k-1}^{(i)}, p_{S,k} \rangle}. \tag{4}$$

Standard Update: If at time k, the predicted multi-target density is multi-Bernoulli of the form

$$\pi_{k|k-1} = \{(r_{k|k-1}^{(i)}, p_{k|k-1}^{(i)})\}_{i=1}^{M_{k|k-1}},$$

then, given a measurement set Z_k, the posterior multi-target density is multi-Bernoulli given by a union of legacy (missed detections) and updated (measurement corrected) components

$$\pi_k \approx \{(r_{L,k}^{(i)}, p_{L,k}^{(i)})\}_{i=1}^{M_{k|k-1}} \cup \{(r_{U,k}(z), p_{U,k}(\cdot; z))\}_{z \in Z_k}, \tag{5}$$

where

$$r_{L,k}^{(i)} = r_{k|k-1}^{(i)} \frac{1 - \langle p_{k|k-1}^{(i)}, p_{D,k} \rangle}{1 - r_{k|k-1}^{(i)} \langle p_{k|k-1}^{(i)}, p_{D,k} \rangle}, \tag{6}$$

$$p_{L,k}^{(i)}(\breve{x}) = p_{k|k-1}^{(i)}(x)\frac{1 - p_{D,k}(\breve{x})}{1 - \langle p_{k|k-1}^{(i)}, p_{D,k}\rangle}, \tag{7}$$

$$r_{U,k}(z) = \frac{\sum_{i=1}^{M_{k|k-1}} \frac{r_{k|k-1}^{(i)}(1-r_{k|k-1}^{(i)})\langle p_{k|k-1}^{(i)}, g_k(z|\cdot)p_{D,k}\rangle}{(1-r_{k|k-1}^{(i)}\langle p_{k|k-1}^{(i)}, p_{D,k}\rangle)^2}}{\kappa_k(z) + \sum_{i=1}^{M_{k|k-1}} \frac{r_{k|k-1}^{(i)}\langle p_{k|k-1}^{(i)}, g_k(z|\cdot)p_{D,k}\rangle}{1-r_{k|k-1}^{(i)}\langle p_{k|k-1}^{(i)}, p_{D,k}\rangle}}, \tag{8}$$

$$p_{U,k}(\breve{x}; z) = \frac{\sum_{i=1}^{M_{k|k-1}} \frac{r_{k|k-1}^{(i)}}{1-r_{k|k-1}^{(i)}} p_{k|k-1}^{(i)}(\breve{x}) g_k(z|\breve{x}) p_{D,k}(\breve{x})}{\sum_{i=1}^{M_{k|k-1}} \frac{r_{k|k-1}^{(i)}}{1-r_{k|k-1}^{(i)}} \langle p_{k|k-1}^{(i)}, g_k(z|\cdot)p_{D,k}\rangle}. \tag{9}$$

Statistically consistent multi-target estimators, such as the Joint Multi-Target (JoM) and Marginal Multi-Target (MaM) estimators [16], can be applied to the multi-target posterior, though these particular estimators can be computationally expensive to compute in practice. An alternative suboptimal procedure, as originally proposed in [16], [28], which is intuitive and computationally efficient is instead reviewed here. This is a two step process which first estimates the number of targets and then the individual target states. The posterior target cardinality can be estimated with either an expected a posteriori (EAP) or maximum a posteriori (MAP) estimate on the posterior cardinality distribution. Once established, the individual target states can be extracted by selecting the relevant number of posterior Bernoulli components with the highest existence probabilities, and computing either the EAP or MAP estimate on the corresponding posterior probability densities for the target state variable. The optimality of this estimator is however still an open question.

III. Adaptation for Unknown Clutter and Detection Profile

The multi-Bernoulli filter for unknown non-homogeneous clutter intensity and detection probability profile can be derived from the original multi-Bernoulli filter with a specially chosen state space. This approach is inspired by that for the CPHD/PHD filters in [17], [18], and is adapted here specifically for parameterized density approximations such as the multi-Bernoulli filter. The underlying idea is explained as follows.

An unknown and non-homogeneous clutter intensity is accommodated by modelling individual clutter returns based on individual clutter targets or generators. Each clutter generator is analogous to an actual target, in the sense that clutter generators have their own separate models for births and deaths as well as transition, likelihood and detections or misses. However, the two types of clutter generators and actual targets are distinct, and cannot evolve into the other type. The intuition here is that the clutter generators will dynamically distribute themselves around the state space to explain the prevailing false alarm conditions. Each of the clutter targets will thus attempt to track the individual false alarms or clutter returns wherever they happen to appear over time. With this model, the filter is, in principle, able to adaptively estimate the clutter rate as well as any type of clutter density. Naturally the filter will incur some delay in estimating the clutter parameters. The proposed filter performs best if the clutter intensity is static or slowly time varying relative to the data rate, since its estimates of the clutter parameters will eventually 'lock' and continue to 'track' the true values.

An unknown and non-homogeneous detection profile is accommodated by incorporating an unknown detection probability into the target state variable. This is achieved by considering an augmented state, given by the usual kinematic state, as well as an augmented variable, which represents a corresponding unknown and state dependent detection probability. The intuition here is that the filter should implicitly estimate the unknown detection probability according to how well the measurements fit the underlying target/object model.

Formally, we proceed as follows. Let $\mathcal{X}^{(\Delta)} = [0, 1]$ denote the space of detection probabilities, $\mathcal{X} = \mathbb{R}^{n_x}$ denote the space for the target kinematics, and $\{0, 1\}$ denote the discrete space of labels for clutter generators and actual targets. The convention that the label $u = 0$ denotes clutter generators and the label $u = 1$ denotes actual targets will be used throughout. The new state space is given by

$$\breve{\mathcal{X}} = \mathcal{X}^{(\Delta)} \times \mathcal{X} \times \{0, 1\},$$

where $\times$ denotes a Cartesian product. Consequently, the state variable takes on the value $\breve{x} = (a, x, u) = (augmentation, kinematics, label)$. The following convention regarding the discrete target label and arbitrary functions defined on the new state space is used throughout

$$f(a, x, u) = f_u(a, x).$$

Since the last term in the Cartesian product $\breve{\mathcal{X}}$ is the discrete space $\{0, 1\}$, integration on $\breve{\mathcal{X}}$ is given by

$$\int_{\breve{\mathcal{X}}} f(\breve{x}) d\breve{x} = \int_{\mathcal{X}^{(\Delta)} \times \mathcal{X}} f_0(a, x) da dx + \int_{\mathcal{X}^{(\Delta)} \times \mathcal{X}} f_1(a, x) da dx.$$

In the following, an explicit specification of the single target models on the new state space is given. Note that clutter generators and actual targets have separate model parameters, and specifically that, clutter generators can never become actual targets and vice-versa. Note also that the standard Poisson clutter term, has accordingly been set to 'zero' with $\kappa_k(z) \equiv 0$, since false alarms are now modelled by dynamic clutter generators. For the specification block below, the single-target models for actual targets and clutter generators are parameterized by the variable u. Setting $u = 1$ yields the complete single target model for actual targets, and setting $u = 0$ yields the complete single target model for clutter generators.

Note that additional subscripts $\mathcal{X}$ and Δ are used to identify individual functions defined on the single target state space $\mathbb{R}^{n_x}$ and the space of detection probabilities $[0,1]$ respectively.

$$\begin{aligned}
p_{S,k}(\alpha,\zeta,u) &= p_{S,u,k}(\zeta) = \textit{probability of survival to time } k, \textit{ given previous state } \zeta, \\
p_{D,k}(a,x,u) &= p_{D,u,k}(a,x) = a = \textit{probability of detection at time } k, \textit{for kinematic state } x, \\
f_{k|k-1}(a,x,u|\alpha,\zeta,u) &= f_{u,k|k-1}(a,x|\alpha,\zeta) = f_{\Delta,u,k|k-1}(a|\alpha) f_{\mathcal{X},u,k|k-1}(x|\zeta) \\
&= \textit{transition density to time } k, \textit{for target state, given previous state } \alpha,\zeta, \\
f_{\Delta,u,k|k-1}(a|\alpha) &= \textit{transition density to time } k, \textit{for detection probability, given previous value } \alpha, \\
f_{\mathcal{X},u,k|k-1}(x|\zeta) &= \textit{transition density to time } k, \textit{for kinematic state, given previous value } \zeta, \\
g_k(z|a,x,u) &= g_{u,k}(z|x) = \textit{likelihood at time } k, \textit{ given current state } x, \\
\{(r^{(i)}_{\Gamma,k}, p^{(i)}_{\Gamma,k}(\cdot,\cdot,u))\}_{i=1}^{M_{\Gamma,k}} &= \{(r^{(i)}_{\Gamma,k}, p^{(i)}_{\Gamma,u,k})\}_{i=1}^{M_{\Gamma,k}} \\
&= \textit{parameters of the multi-Bernoulli RFS of births at time } k.
\end{aligned}$$

The prediction and update steps of the proposed multi-Bernoulli filter follow as a direct consequence of applying the new state space and new single target models to the standard multi-Bernoulli filter.

A. Recursion

Proposition: (Prediction) If at time $k-1$, the posterior multi-target density is multi-Bernoulli of the form

$$\pi_{k-1} = \{(r^{(i)}_{k-1}, p^{(i)}_{u,k-1})\}_{i=1}^{M_{k-1}},$$

then the predicted multi-target density is multi-Bernoulli given by a union of birth and persisting or predicted components

$$\pi_{k|k-1} = \{(r^{(i)}_{\Gamma,k}, p^{(i)}_{\Gamma,u,k})\}_{i=1}^{M_{\Gamma,k}} \cup \{(r^{(i)}_{P,k|k-1}, p^{(i)}_{P,u,k|k-1})\}_{i=1}^{M_{k-1}}, \tag{10}$$

where

$$r^{(i)}_{P,k|k-1} = r^{(i)}_{k-1} \textstyle\sum_{u=0,1} \langle p^{(i)}_{u,k-1}, p_{S,u,k}\rangle, \tag{11}$$

$$p^{(i)}_{P,u,k|k-1}(a,x) = \frac{\langle f_{u,k|k-1}(a,x|\cdot,\cdot), p^{(i)}_{u,k-1} p_{S,u,k}\rangle}{\langle p^{(i)}_{u,k-1}, p_{S,u,k}\rangle}. \tag{12}$$

Proof: We begin by substituting $p_{S,k}(\alpha,\zeta,u) = p_{S,u,k}(\zeta)$, $f_{k|k-1}(a,x,u|\alpha,\zeta,u) = f_{u,k|k-1}(a,x|\alpha,\zeta)$, and the birth model $\{(r^{(i)}_{\Gamma,k}, p^{(i)}_{\Gamma,k}(\cdot,\cdot,u))\}_{i=1}^{M_{\Gamma,k}} = \{(r^{(i)}_{\Gamma,k}, p^{(i)}_{\Gamma,u,k}(\cdot,\cdot))\}_{i=1}^{M_{\Gamma,k}}$, into the standard multi-Bernoulli prediction (2)-(4) but with the new state space $\breve{\mathcal{X}} = \mathcal{X}^{(\Delta)} \times \mathcal{X} \times \{0,1\}$. The general form of the predicted density (10) follows from (2). The expression for the probability of existence (11) of surviving tracks, follows from applying the definition for integrals on the new state space to (3), which results in a sum over all target labels $u = 0, 1$. The expression for the probability densities (12) of surviving tracks, parameterized by the variable $u = 0, 1$, corresponding to clutter generators and actual targets respectively, follows from (4) by noting that the inner products are now defined by integrals over a and x. The birth model $\{(r_{\Gamma,k}^{(i)}, p_{\Gamma,k}^{(i)}(\cdot,\cdot,u))\}_{i=1}^{M_{\Gamma,k}}$ is substituted in directly and requires no manipulation.

Remark: Similar to the prediction step in the standard Multi-Bernoulli filter, the prediction step for the proposed filter involves separate terms for new birth tracks (subscript Γ) and existing track predictions (subscript P), except that there are separate components for actual targets ($u = 1$) and clutter generators ($u = 0$). The birth terms feature independently, whereas the prediction terms involve the standard single target prediction, weighted and normalized by the survival probability.

Proposition: (Update) If at time k, the predicted multi-target density is multi-Bernoulli of the form

$$\pi_{k|k-1} = \{(r_{k|k-1}^{(i)}, p_{u,k|k-1}^{(i)})\}_{i=1}^{M_{k|k-1}},$$

then, given a measurement set Z_k, the posterior multi-target density is multi-Bernoulli given by a union of legacy (missed detection) and updated (measurement corrected) components

$$\pi_k = \{(r_{L,k}^{(i)}, p_{L,u,k}^{(i)})\}_{i=1}^{M_{k|k-1}} \cup \{(r_{U,k}(z), p_{U,u,k}(\cdot; z))\}_{z\in Z_k}, \tag{13}$$

where for $u = 0, 1$

$$r_{L,k}^{(i)} = \textstyle\sum_{u=0,1} r_{L,u,k}^{(i)}, \tag{14}$$

$$r_{L,u,k}^{(i)} = \frac{r_{k|k-1}^{(i)} \langle p_{u,k|k-1}^{(i)}, 1 - p_{D,u,k}\rangle}{1 - r_{k|k-1}^{(i)} \sum_{u'=0,1} \langle p_{u',k|k-1}^{(i)}, p_{D,u',k}\rangle}, \tag{15}$$

$$p_{L,u,k}^{(i)}(a,x) = \frac{(1-a) p_{u,k|k-1}^{(i)}(a,x)}{\sum_{u'=0,1} \langle p_{u',k|k-1}^{(i)}, 1 - p_{D,u',k}\rangle}, \tag{16}$$

$$r_{U,k}(z) = \textstyle\sum_{u=0,1} r_{U,u,k}(z), \tag{17}$$

$$r_{U,u,k}(z) = \frac{\sum_{i=1}^{M_{k|k-1}} \frac{r_{k|k-1}^{(i)}(1-r_{k|k-1}^{(i)}) \langle p_{u,k|k-1}^{(i)}, g_{u,k}(z|\cdot) p_{D,u,k}\rangle}{(1-r_{k|k-1}^{(i)} \sum_{u'=0,1} \langle p_{u',k|k-1}^{(i)}, p_{D,u',k}\rangle)^2}}{\sum_{i=1}^{M_{k|k-1}} \frac{r_{k|k-1}^{(i)} \sum_{u'=0,1} \langle p_{u',k|k-1}^{(i)}, g_{u',k}(z|\cdot) p_{D,u',k}\rangle}{1-r_{k|k-1}^{(i)} \sum_{u'=0,1} \langle p_{u',k|k-1}^{(i)}, p_{D,u',k}\rangle}}, \tag{18}$$

$$p_{U,u,k}(a,x;z) = \frac{\sum_{i=1}^{M_{k|k-1}} \frac{r_{k|k-1}^{(i)}}{1-r_{k|k-1}^{(i)}} p_{u,k|k-1}^{(i)}(a,x) g_{u,k}(z|x) \cdot a}{\sum_{u'=0,1} \sum_{i=1}^{M_{k|k-1}} \frac{r_{k|k-1}^{(i)}}{1-r_{k|k-1}^{(i)}} \langle p_{u',k|k-1}^{(i)}, g_{u',k}(z|\cdot) p_{D,u',k}\rangle}. \tag{19}$$

Proof: We begin by substituting $p_{D,k}(a,x,u) = p_{D,u,k}(a,x)$, $g_k(z|a,x,u) = g_{u,k}(z|x)$, into the standard multi-Bernoulli update (5)-(9) but with the new state space $\breve{\mathcal{X}} = \mathcal{X}^{(\Delta)} \times \mathcal{X} \times \{0,1\}$ and set the standard Poisson clutter rate $\kappa_k(z) = 0$. Note that the legacy or missed terms are indicated by the subscript L, and the data or measurement updated terms are indicated by the subscript U. The expressions (14) and (17) for $r_{L,k}^{(i)}$ and $r_{U,k}(z)$, respectively, follow from applying the definition for integrals on the new state space to (6) and (8). This results in a sum over all target labels, and produces $r_{L,k}^{(i)} = r_{L,0,k}^{(i)} + r_{L,1,k}^{(i)}$, as well as $r_{U,k}(z) = r_{U,0,k}(z) + r_{U,1,k}(z)$. The respective components, $r_{L,0,k}^{(i)}$, $r_{L,1,k}^{(i)}$ and $r_{U,0,k}(z)$, $r_{U,1,k}(z)$ are obtained by separating out the contributions from $u = 0$ and $u = 1$ in the top most numerator of (6) and (8), respectively. The normalizing constants in (15) and (18) again follow from the definition of integrals on the new state space. The expressions (16) and (19) for the probability densities $p_{L,u,k}^{(i)}$ and $p_{U,u,k}(z)$, parameterized by the variable $u = 0, 1$, follow directly from (7) and (9), where the normalizing constant is evaluated via the definition of integrals on the new state space and consequently expands to a sum over $u =$ 0,1.

Remark: Similar to the update step in the standard Multi-Bernoulli filter, the update step for the proposed filter involves separate terms for both legacy tracks (subscript L where there is no corresponding measurement correction) and updated tracks (subscript U where each term corresponds to a single measurement correction), except that there are separate components for actual targets ($u = 1$) and clutter generators ($u = 0$). Each surviving and new born track induces a legacy track, and each $z \in Z_k$ induces a measurement updated track involving a sum over all predicted tracks. The expressions for the legacy and updated tracks can be interpreted as pseudo single target updates which simultaneously weight and normalize for the unknown clutter rate and detection profile.

Remark: The proposed adaptation for unknown clutter intensity and detection probability has a generally similar but slightly higher complexity compared to the standard Multi-Bernoulli filter. This is because the underlying structure of the filter is essentially unchanged. In practice and depending on the particular scenario, the proposed adaption may require many more components to be maintained and propagated. This is to accommodate the additional uncertainty in the problem.

B. State Estimation

As with the standard Multi-Bernoulli filter, statistically consistent multi-target estimators such as the Joint Multi-Target (JoM) and Marginal Multi-Target (MaM) estimators [16] can be applied, however these can be difficult and computationally expensive to compute. We instead adapt, for the proposed filter, the cheaper but suboptimal two step estimation procedure originally described for standard Multi-Bernoulli filter. Notice that since the target state describes both actual targets and clutter generators, care must be taken to distinguish the target types in performing state extraction.

The update equations (14)-(19) also imply that the posterior multi-Bernoulli parameters $\{(r_k^{(i)}, p_k^{(i)})\}_{i=1}^{M_k}$, are decomposable into $r_k^{(i)} = r_{0,k}^{(i)} + r_{1,k}^{(i)}$, $p_{0,k}^{(i)}(a,x) = p_k^{(i)}(a,x,0)$ and $p_{1,k}^{(i)}(a,x) = p_k^{(i)}(a,x,1)$. A MAP estimate on the posterior cardinality distribution is not appropriate, since the posterior cardinality distribution implicitly computed by the filter includes both actual targets and clutter generators. Hence, the number of actual targets must be estimated as an EAP estimate by considering only the parts of the existence probabilities which pertain to actual targets with i.e. $\hat{N}_k = \sum_{i=1}^{M_k} r_{1,k}^{(i)}$. An estimate of the variance on the target number estimate is $\hat{\sigma}_{N,k}^2 = \sum_{i=1}^{M_k} r_{1,k}^{(i)}(1 - r_{1,k}^{(i)})$. Extraction of individual target states must then be computed as either an EAP or MAP estimator on the relevant Bernoulli components, by considering the corresponding probability density $p_{1,k}^{(i)}(a,x)$, where the augmented parameter can be treated as a nuisance variable and integrated out, or reported as an estimate of the detection probability a at the location x. The mean rate of clutter can be simply estimated as $\hat{\lambda}_{c,k} = \sum_{i=1}^{M_k} r_{0,k}^{(i)} \int a p_{0,k}^{(i)}(a,x) da dx$, which is the expected number of detections produced by all clutter generators, i.e. the expected number of false alarms.

IV. Particle Implementation

A particle or Sequential Monte Carlo implementation of the proposed multi-Bernoulli filter follows directly from that for the standard multi-Bernoulli filter [28]. Since the latter is a direct adaption of standard particle or SMC methods, convergence results for the implementation of the proposed multi-Bernoulli filter also follow, see for example the convergence results in [13]. The prediction and update steps are given below.

A. Recursion

If at time $k-1$, the (multi-Bernoulli) posterior multi-target density

$$\pi_{k-1} = \{(r_{k-1}^{(i)}, p_{u,k-1}^{(i)})\}_{i=1}^{M_{k-1}} \tag{20}$$

is given, for $i = 1, ..., M_{k-1}$ and for $u = 0, 1$

$$p_{u,k-1}^{(i)}(a,x) = \sum_{j=1}^{L_{u,k-1}^{(i)}} w_{u,k-1}^{(i,j)} \delta_{(a_{u,k-1}^{(i,j)}, x_{u,k-1}^{(i,j)})}(a,x), \tag{21}$$

then given importance (or proposal) densities $q_{u,k}^{(i)}(\cdot,\cdot|a_{u,k-1}, x_{u,k-1}, Z_k)$, and $b_{u,k}^{(i)}(\cdot|Z_k)$ with support (i.e. points where the function is non-zero) satisfying

$$\textit{support}(p_{u,k-1}^{(i)}) \quad \subseteq \quad \textit{support}(q_{u,k}^{(i)}),$$

$$support(p^{(i)}_{\Gamma,u,k}) \quad \subseteq \quad support(b^{(i)}_{u,k}),$$

the predicted (multi-Bernoulli) multi-target density

$$\pi_{k|k-1} = \{(r^{(i)}_{\Gamma,k}, p^{(i)}_{\Gamma,u,k})\}_{i=1}^{M_{\Gamma,k}} \cup \{(r^{(i)}_{P,k|k-1}, p^{(i)}_{P,u,k|k-1})\}_{i=1}^{M_{k-1}} \tag{22}$$

can be computed as follows

$$r^{(i)}_{P,k|k-1} = r^{(i)}_{k-1} \sum_{u=0,1} \sum_{j=1}^{L^{(i)}_{u,k-1}} w^{(i,j)}_{u,k-1} p_{S,u,k}(x^{(i,j)}_{u,k-1}), \tag{23}$$

$$p^{(i)}_{P,u,k|k-1}(a,x) = \sum_{j=1}^{L^{(i)}_{u,k-1}} \tilde{w}^{(i,j)}_{P,u,k|k-1} \delta_{a^{(i,j)}_{P,u,k|k-1}, x^{(i,j)}_{P,u,k|k-1}}(a,x), \tag{24}$$

$$r^{(i)}_{\Gamma,k} = \textit{parameter given by birth model,} \tag{25}$$

$$p^{(i)}_{\Gamma,u,k}(a,x) = \sum_{j=1}^{L^{(i)}_{\Gamma,k}} \tilde{w}^{(i,j)}_{\Gamma,u,k} \delta_{a^{(i,j)}_{\Gamma,u,k} x^{(i,j)}_{\Gamma,u,k}}(a,x), \tag{26}$$

where for $u = 0, 1$

$$a^{(i,j)}_{P,u,k|k-1}, x^{(i,j)}_{P,u,k|k-1} \sim q^{(i)}_{u,k}(\cdot | a^{(i,j)}_{u,k-1}, x^{(i,j)}_{u,k-1}, Z_k), \;\; j = 1, ..., L^{(i)}_{u,k-1}$$

$$w^{(i,j)}_{P,u,k|k-1} = \frac{w^{(i,j)}_{u,k-1} f_{\Delta,u,k|k-1}(a^{(i,j)}_{P,u,k|k-1} | a^{(i,j)}_{u,k-1}) f_{\mathcal{X},u,k|k-1}(x^{(i,j)}_{P,u,k|k-1} | x^{(i,j)}_{u,k-1}) p_{S,u,k}(x^{(i,j)}_{k-1})}{q^{(i)}_{u,k}(a^{(i,j)}_{P,u,k|k-1}, x^{(i,j)}_{P,u,k|k-1} | a^{(i,j)}_{u,k-1}, x^{(i,j)}_{u,k-1}, Z_k)},$$

$$\tilde{w}^{(i,j)}_{P,u,k|k-1} = w^{(i,j)}_{P,u,k|k-1} / \left(\sum_{u'=0,1} \sum_{j'=1}^{L^{(i)}_{u',k-1}} w^{(i,j')}_{P,u',k|k-1} \right),$$

$$x^{(i,j)}_{\Gamma,u,k} \sim b^{(i)}_{u,k}(\cdot | Z_k), \;\; j = 1, ..., L^{(i)}_{\Gamma,u,k}$$

$$w^{(i,j)}_{\Gamma,u,k} = \frac{p^{(i)}_{\Gamma,u,k}(a^{(i,j)}_{\Gamma,u,k}, x^{(i,j)}_{\Gamma,u,k})}{b^{(i)}_{u,k}(a^{(i,j)}_{\Gamma,u,k}, x^{(i,j)}_{\Gamma,u,k} | Z_k)},$$

$$\tilde{w}^{(i,j)}_{\Gamma,u,k} = w^{(i,j)}_{\Gamma,u,k} / \left(\sum_{u'=0,1} \sum_{j'=1}^{L^{(i)}_{\Gamma,u',k}} w^{(i,j')}_{\Gamma,u',k} \right).$$

Equation (22) above is the same as original prediction (10), while equations (23)-(24) are the particle approximations of the predicted components (11)-(12), and equations (25)-(26) are sampled approximations for the birth densities in (10).

If at time k, the predicted (multi-Bernoulli) multi-target density

$$\pi_{k|k-1} = \{(r^{(i)}_{k|k-1}, p^{(i)}_{k|k-1})\}_{i=1}^{M_{k|k-1}} \tag{27}$$

is given, and each $p^{(i)}_{u,k|k-1}$ for $i = 1, ..., M_{k|k-1}$ and $u = 0, 1$ is represented by

$$p^{(i)}_{u,k|k-1}(a,x) = \sum_{j=1}^{L^{(i)}_{u,k|k-1}} w^{(i,j)}_{u,k|k-1} \delta_{(a^{(i,j)}_{u,k|k-1}, x^{(i,j)}_{u,k|k-1})}(a,x), \tag{28}$$

then, given a measurement set Z_k, the updated (multi-Bernoulli) multi-target density

$$\pi_k = \{(r^{(i)}_{L,k}, p^{(i)}_{L,u,k})\}_{i=1}^{M_{k|k-1}} \cup \{(r_{U,k}(z), p_{U,u,k}(\cdot; z))\}_{z \in Z_k} \tag{29}$$

can be computed as follows

$$
\begin{aligned}
r_{L,k}^{(i)} &= \sum_{u=0,1} r_{L,u,k}^{(i)}, && (30)\\
r_{L,u,k}^{(i)} &= r_{k|k-1}^{(i)} \frac{\bar{\varrho}_{L,u,k}^{(i)} - \varrho_{L,u,k}^{(i)}}{1 - r_{k|k-1}^{(i)} \sum_{u'=0,1} \varrho_{L,u',k}^{(i)}}, && (31)\\
p_{L,u,k}^{(i)}(a,x) &= \sum_{j=1}^{L_{u,k|k-1}^{(i)}} \tilde{w}_{L,u,k}^{(i,j)} \delta_{a_{u,k|k-1}^{(i,j)} x_{u,k|k-1}^{(i,j)}}(a,x), && (32)\\
r_{U,k}(z) &= \sum_{u=0,1} r_{U,u,k}(z), && (33)\\
r_{U,u,k}(z) &= \frac{\sum_{i=1}^{M_{k|k-1}} \frac{r_{k|k-1}^{(i)}(1-r_{k|k-1}^{(i)})\varrho_{U,u,k}^{(i)}(z)}{(1-r_{k|k-1}^{(i)}\sum_{u'=0,1}\varrho_{L,u',k}^{(i)})^2}}{\sum_{i=1}^{M_{k|k-1}} \frac{r_{k|k-1}^{(i)}\sum_{u'=0,1}\varrho_{U,u',k}^{(i)}(z)}{1-r_{k|k-1}^{(i)}\sum_{u'=0,1}\varrho_{L,u',k}^{(i)}}}, && (34)\\
p_{U,u,k}(a,x;z) &= \sum_{i=1}^{M_{k|k-1}} \sum_{j=1}^{L_{u,k|k-1}^{(i)}} \tilde{w}_{U,u,k}^{(i,j)}(z) \delta_{a_{u,k|k-1}^{(i,j)} x_{u,k|k-1}^{(i,j)}}(a,x), && (35)
\end{aligned}
$$

where for $u = 0, 1$

$$
\begin{aligned}
\bar{\varrho}_{L,u,k}^{(i)} &= \sum_{j=1}^{L_{u,k|k-1}^{(i)}} w_{u,k|k-1}^{(i,j)},\\
\varrho_{L,u,k}^{(i)} &= \sum_{j=1}^{L_{u,k|k-1}^{(i)}} w_{u,k|k-1}^{(i,j)} a_{u,k|k-1}^{(i,j)},\\
\tilde{w}_{L,u,k}^{(i,j)} &= w_{L,u,k}^{(i,j)} / \left(\sum_{u'=0,1} \sum_{j'=1}^{L_{u',k|k-1}^{(i)}} w_{L,u',k}^{(i,j')} \right),\\
w_{L,u,k}^{(i,j)} &= w_{u,k|k-1}^{(i,j)}(1 - a_{u,k|k-1}^{(i,j)}),\\
\varrho_{U,u,k}^{(i)}(z) &= \sum_{j=1}^{L_{u,k|k-1}^{(i)}} w_{u,k|k-1}^{(i,j)} g_{u,k}(z|x_{u,k|k-1}^{(i,j)}) a_{u,k|k-1}^{(i,j)},\\
\tilde{w}_{U,u,k}^{(i,j)}(z) &= w_{U,u,k}^{(i,j)}(z) / \left(\sum_{u'=0,1} \sum_{i'=1}^{M_{k|k-1}} \sum_{j'=1}^{L_{u',k|k-1}^{(i')}} w_{U,u',k}^{(i',j')}(z) \right),\\
w_{U,u,k}^{(i,j)}(z) &= \frac{r_{k|k-1}^{(i)}}{1 - r_{k|k-1}^{(i)}} w_{u,k|k-1}^{(i,j)} g_{u,k}(z|x_{u,k|k-1}^{(i,j)}) a_{u,k|k-1}^{(i,j)}.
\end{aligned}
$$

Equation (29) above is the same as original update (13), while equations (30)-(32) are the particle approximations of the legacy components (14)-(16), and equations (33)-(35) are the particle approximations of the updated components (17)-(19).

Remark: The complexity of proposed adaptation is generally higher than that of the standard Multi-Bernoulli filter. This is because the proposed filter maintains individual clutter generator which 'track' the individual false alarms. The standard Multi-Bernoulli filter specifies an a priori model for clutter in the usual form of a Poisson RFS. However both the proposed adaptation and the standard filter use the same mechanism for tracking actual targets. The proposed solution is effectively a strategy for running a bank of parallel particle filters, where each particle filter corresponds to separate Bernoulli components, which collectively attempt to track actual targets and clutter generators. In high signal-to-noise ratio scenarios with low clutter rate and high detection probability, the complexity of the proposed adaptation is slightly higher than in the standard formulation.

B. *Implementation Issues and State Estimation*

The number of Bernoulli components or tracks required to represent the posterior density increases with time, due to the birth of objects in the prediction, and the averaging of predicted components in the update. To reduce the Bernoulli components or tracks, at each time step, pruning is performed by discarding those with existence probabilities below a threshold $T_{\min}$ (e.g. 10^{-4}).

For the same reason, the number of particles required also increases over time. It is then desirable to allocate the total number of particles to be proportional to the expected number of targets present. This can be achieved by resampling which also helps to mitigate degeneracy, Thus, at each time step, the number of particles allocated to each Bernoulli component is set in proportion to its probability of existence, i.e. for the prediction use importance sampling to draw $L_{\Gamma,k}^{(i)} = \max(r_{\Gamma,k}^{(i)} L_{\max}, L_{\min})$ particles per birth term, and post update resample $L_k^{(i)} = \max(r_k^{(i)} L_{\max}, L_{\min})$ particles for each updated Bernoulli component. Note that a maximum of $L_{\max}$ and minimum of $L_{\min}$ number of particles per Bernoulli component or track should also be enforced to maintain tractability and consistency. Any resampling procedure can be used, including the standard multinomial technique.

State estimation can be performed as previously outlined using the particle approximations to the multi-target density. The estimated number of targets is the posterior mean $\hat{N}_k = \sum_{i=1}^{M_k} r_{1,k}^{(i)}$. Individual state estimates $m_k^{(1)}, ..., m_k^{(\hat{N}_k)}$ are the means of the corresponding Bernoulli components or tracks $m_k^{(i)} = \sum_{j=1}^{L_k^{(i)}} w_{1,k}^{(i,j)} x_{1,k}^{(i,j)}$. The estimated rate of clutter is $\hat{\lambda}_{c,k} = \sum_{i=1}^{M_k} r_{0,k}^{(i)} \sum_{j=1}^{L_k^{(i)}} w_{0,k}^{(i,j)} a_{0,k}^{(i,j)}$.

V. Numerical Studies

A non-linear multi-target tracking scenario is used to demonstrate the performance of the proposed multi-Bernoulli filter via the proposed particle implementation. A total of 10 targets appear on the scene throughout the scenario. Target tracks are shown in Figure 1 on the half disc of radius $2000m$ with the start and stop positions of each track. The target state $\breve{x}_k = [\ a_k, p_{x,k}, \dot{p}_{x,k}, p_{y,k}, \dot{p}_{y,k}, \omega_k, u_k\ ]^T$ comprises the the unknown detection probability a_k, the planar position and velocity $\ddot{x}_k = [\ p_{x,k}, \dot{p}_{x,k}, p_{y,k}, \dot{p}_{y,k}\ ]^T$, the turn rate ω_k, and the type label $u_k \in \{0, 1\}$. Sensor returns are bearings and range vectors of the form $z_k = [\ \theta_k, r_k\ ]^T$. The model parameters are given below where the notation $\mathcal{N}(\cdot; m, P)$ is used to denote a Gaussian density with mean m and covariance P, and the notation $\beta(\cdot; s, t)$ is used to denote a

Beta density with parameters s, t. The Gaussian density is used to model the kinematic and observation noise. The Beta density is used to model the unknown detection probability, since the density covers the range $[0, 1]$, and has sufficient flexibility to capture various detection probability profiles.

A. Model for Actual Targets

Actual targets follow a coordinated turn model, with the following transition density for the kinematic state $x_k = [\ddot{\dot{x}}_k, \omega_k]$

$$f_{\mathcal{X},1,k|k-1}(x_k|x_{k-1}) = \mathcal{N}(x_k; m_{x,1,k|k-1}(x_{k-1}), P_{x,1,k|k-1}),$$

where $m_{x,1,k|k-1}(x_{k-1}) = [F(\omega_{k-1})\ddot{\dot{x}}_{k-1}, \omega_{k-1}]^T$, $P_{x,1,k|k-1} = \mathrm{diag}([\sigma_w^2 GG^T, \sigma_\omega^2])$,

$$F(\omega) = \begin{bmatrix} 1 & \frac{\sin\omega T}{\omega} & 0 & -\frac{1-\cos\omega T}{\omega} \\ 0 & \cos\omega T & 0 & -\sin\omega T \\ 0 & \frac{1-\cos\omega T}{\omega} & 1 & \frac{\sin\omega T}{\omega} \\ 0 & \sin\omega T & 0 & \cos\omega T \end{bmatrix}, G = \begin{bmatrix} \frac{T^2}{2} & 0 \\ T & 0 \\ 0 & \frac{T^2}{2} \\ 0 & T \end{bmatrix},$$

and $T = 1s$ is the sampling time, $\sigma_w = 15m/s^2$ is the standard deviation of the process noise, $\sigma_\omega = 3\pi/180 rad/s$ is the standard deviation of the turn rate noise. The transition density for the probability of detection variable is

$$f_{\Delta,1,k|k-1}(a_k|a_{k-1}) = \beta(a_k; s_{1,k|k-1}, t_{1,k|k-1}),$$

where $s_{1,k|k-1} = \Big(\mu_{a,1,k|k-1}(1 - \mu_{a,1,k|k-1})/[\sigma_{a,1,k|k-1}]^2 - 1\Big)\mu_{a,1,k|k-1}$ and $t_{1,k|k-1} = \Big(\mu_{a,1,k|k-1}(1 - \mu_{a,1,k|k-1})/[\sigma_{a,1,k|k-1}]^2 - 1\Big)(1 - \mu_{a,1,k|k-1})$ gives a chosen (matching) mean of $\mu_{a,1,k|k-1} = a_{k-1}$ and a chosen (a priori) standard deviation of $\sigma_{a,1,k|k-1} = 0.01$ for this transition. The label variable is fixed at $u_k \equiv 1$ for actual targets. The survival probability for actual targets is $p_{S,1,k}(x_k) = 0.99$. Actual targets produce noisy bearings and range measurements $z_k = [\ \theta_k, r_k\ ]^T$ with likelihood given by

$$g_{1,k}(z_k|x_k) = \mathcal{N}(z_k; m_{z,1,k}(x_k), P_{z,1,k}),$$

where $m_{z,1,k} = [\arctan(p_{x,k}/p_{y,k}), \sqrt{p_{x,k}^2 + p_{y,k}^2}]$ and $P_{z,1,k} = \mathrm{diag}([\ \sigma_\theta^2, \sigma_r^2\ ]^T)$ with $\sigma_\theta = (\pi/180) rad$ and $\sigma_r = 5m$. Note that the probability of detection for actual targets is unknown to the filter, but the measurement data is simulated according to a state dependent detection probability, which peaks at value of $p_{D,1,k} = 0.98$ at the origin and tapers off to a value of $p_{D,1,k} = 0.92$ at the edge of the surveillance region. The actual target birth process has probability density $\pi_{\Gamma,1,k} = \{(r_{\Gamma,k}^{(i)}, p_{\Gamma,1,k}^{(i)})\}_{i=1}^4$ where $r_{\Gamma,k}^{(1)} = r_{\Gamma,k}^{(2)} = 0.02$, $r_{\Gamma,k}^{(3)} = r_{\Gamma,k}^{(4)} = 0.03$, $p_{\Gamma,1,k}^{(i)}(a_k, x_k) = \beta(a; s_{\Gamma,1,k}^{(i)}, t_{\Gamma,1,k}^{(i)})\mathcal{N}(x; m_{\Gamma,1,k}^{(i)}, P_{\Gamma,1,k}^{(i)})$,

$s_{\Gamma,1,k}^{(i)} = 98$, $t_{\Gamma,1,k}^{(i)} = 2$, $m_{\Gamma,1,k}^{(1)} = [\ -1500, 0, 250, 0, 0\ ]^T$, $m_{\Gamma,1,k}^{(2)} = [\ -250, 0, 1000, 0, 0\ ]^T$, $m_{\Gamma,1,k}^{(3)} = [\, 250, 0, 750, 0, 0\,]^T$, $m_{\Gamma,1,k}^{(4)} = [\, 1000, 0, 1500, 0, 0\,]^T$, and $P_{\Gamma,1,k}^{(i)} = \mathrm{diag}([\, 90,\ 90,\ 90,\ 90,\ 9(\pi/180)\,]^T)^2$.

B. Model for Clutter Generators

Clutter generators are modelled only by their (unknown) detection probability a_k and positions $\ddot{x}_k = [\ p_{x,k}, p_{y,k}\ ]^T$, while their velocities and turn rate are ignored. The positions of clutter generators follow a random walk model given by the transition density $f_{\mathcal{X},0,k|k-1}(\ddot{x}_k|\ddot{x}_{k-1}) = \mathcal{N}(\ddot{x}_k; \ddot{x}_{k-1}, P_{x,0,k|k-1})$ where $P_{x,0,k|k-1} = \mathrm{diag}([\sigma_x^2, \sigma_y^2])$ and $\sigma_x = 1000m$ and $\sigma_y = 500m$ are the noise standard deviations on each axis. The transition density for the augmented part of the state $f_{\Delta,0,k|k-1}(a_k|a_{k-1})$ is given by a Beta density, similar to the expression for actual targets, except with a standard deviation of $\sigma_{a,0,k|k-1} = 0.07$. The label variable is fixed at $u_k \equiv 0$ for clutter generators. The survival probability for clutter generators is $p_{S,0,k}(\ddot{x}_k) = 0.90$. Clutter generators also produce noisy bearings and range measurements with likelihood $g_{0,k}(z_k|x_k)$, similar to the equation for actual targets, except with noise standard deviations $\sigma_\theta = 20(\pi/180)rad$ and $\sigma_r = 400m$. The probability of detection for clutter generators is similarly unknown to the filter. The clutter generator birth process has probability density $\pi_{\Gamma,0,k} = \{(r_{\Gamma,k}^{(i)}, p_{\Gamma,0,k}^{(i)})\}_{i=5}^{24}$ where $r_{\Gamma,k}^{(i)} = 0.1$, $p_{\Gamma,0,k}^{(i)}(a_k, \ddot{x}_k) = \beta(a_k; s_{\Gamma,0,k}^{(i)}, t_{\Gamma,0,k}^{(i)})\mathcal{U}(\ddot{x}_k)$ and $s_{\Gamma,0,k}^{(i)} = t_{\Gamma,0,k}^{(i)} = 5$, $\mathcal{U}(\cdot) = \mathrm{uniform}$. The number of clutter returns is simulated according to a Binomial distribution, with 20 generators and probability 0.5, giving an average of 10 returns per scan, and the spatial distribution of clutter is simulated such that it is increasingly concentrated near the origin and increasingly sparse moving radially outwards (read non-uniform in xy, but uniform in $r\theta$).

C. Filter Parameters

We use a maximum of $L_{\max} = 1000$ and minimum of $L_{\min} = 100$ particles per Bernoulli component or track. The actual number of particles in each component or track is allocated proportional to its existence probability. For both actual targets and clutter generators, their respective transition densities are used as proposals in the prediction step. To generate new samples representing spontaneous births of actual targets, the birth density itself is used as the proposal since it can be easily sampled from. To generate new samples representing spontaneous births of clutter generators, a special measurement driven proposal is used based on the approach in [22]. The measurement driven proposal essentially generates new particles around the location of the measurements, so as to minimize wastage of particles, but weights them in special a way to retain the uniform distribution of the birth density. Track pruning is performed with a threshold of $T_{\min} = 10^{-4}$. The filter is initialized with zero actual targets and with uniformly distributed clutter generators.

D. Results

The filter output for a single run is shown in Figure 2 giving the x and y coordinates of the true and estimated positions, along with the x and y coordinates of the received measurements versus time. It can be seen that the filter has reasonable performance, generally initiating and terminating each of the tracks within several time steps, and generally producing accurate estimates of the target positions. Occasional incidences of false and dropped tracks are observed, although this is to be expected, since the clutter intensity and detection probability are not known and must be dynamically estimated. It is noted though that generally speaking, a faster sampling and data rate should produce improved results with less false and dropped tracks.

The estimated clutter rate and cardinality statistics over 1000 Monte Carlo trials are shown in Figures 3 and 4 respectively. The estimate of the clutter rate appears to be close to the true value. These results confirm that the filter produces reasonable estimates of the number of targets with some lag in initiating and terminating tracks. It is apparent however that the results are generally better during the first half of the simulation but visibly worse during the second half of the simulation where the filter develops a positive bias in the cardinality estimate and a corresponding negative bias in the clutter rate estimate. This is most likely because during the first half, there is a low number of targets which are reasonably well separated, whereas in the second half, the number of targets is higher and the targets themselves are much closer together. These results indicate that the proposed filter still has some difficulty handling closely spaced targets and resolving slow target crossings.

The OSPA miss distance [23] for $p = 1$ and $c = 300m$ is shown versus time in Figure 5, along with its localization and cardinality error components in Figure 6. Comparisons with the standard multi-Bernoulli filter are also shown, where the filter is given the correct clutter and detection model. These results confirm the previous observations, and further indicate as expected, that the newly proposed filter cannot outperform the standard filter which is supplied with the correct clutter and detection model. The magnitude of the performance difference is of the order of 30-35%.

Comparisons with SMC implementations of the PHD and CPHD filters for unknown clutter and detection parameters from [19] are also shown. Note that unlike the standard PHD and CPHD filters, their counterparts for filtering with unknown parameters are expected to have similar performance, due to modelling of clutter with independent generators (see [19] for further explanation). The comparisons indicate that the PHD and CPHD versions perform reasonably well during the first half of the simulation when the target numbers and target density are low. When the target numbers and target density are higher as in the second half of the simulation, the PHD and CPHD versions perform drastically worse due to their reliance on clustering. The proposed multi-Bernoulli filter is more robust in this case since it is better able to cope with non-linear systems.

VI. Conclusion

Estimating clutter rate and detection profile are difficult problems in practice and the capability of multi-target filters to adaptively learn these model parameters is very important. It has been shown that mismatches in clutter and detection parameters can be accommodated by the multi-Bernoulli filter. This solution accommodates non-linear target models as well as a non-homogeneous clutter intensity and detection probability. Numerical studies show that the proposed technique can correct for discrepancies in these parameters with promising results. Compared to the standard multi-Bernoulli filter, the proposed filter does not require a Poisson clutter model. Due to the modelling of clutter based on multiple independent generators, the proposed filter generally assumes a more precise specification of the cardinality distribution of the clutter, as opposed to the standard Poisson distribution. Typically this results in a clutter cardinality distribution which is approximately Binomial when each generator has a similar probability of existence. Consequently, the proposed filter is comparatively more tolerant to high clutter, since it generally assumes a smaller variance on the clutter cardinality distribution. However, the proposed filter is comparatively more sensitive to low probability of detection, since it does not have prior knowledge of the detection profile and must simultaneously estimate this while filtering.

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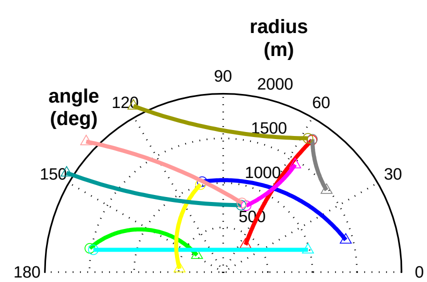

Fig. 1. Target trajectories in polar coordinates. Start/Stop positions for each track are shown with ○/△.

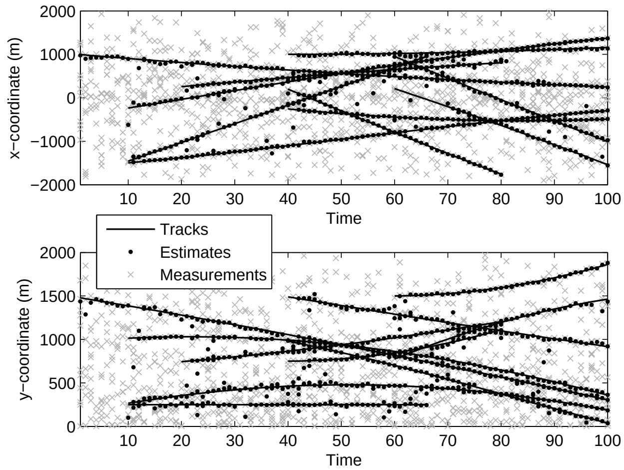

Fig. 2. Proposed multi-Bernoulli filter: estimates and tracks in x and y coordinates versus time

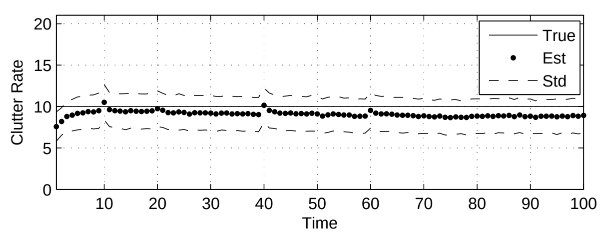

Fig. 3. Monte Carlo average results for multi-Bernoulli filter with unknown parameters: True and estimated clutter rates.

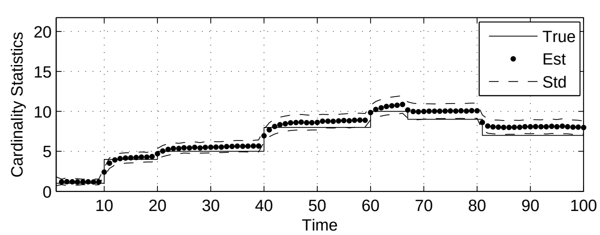

Fig. 4. Monte Carlo average results for multi-Bernoulli filter with unknown parameters: True and estimated target numbers.

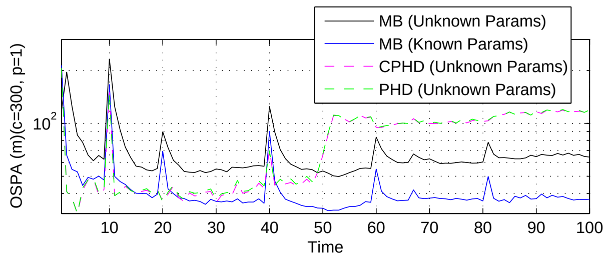

Fig. 5. OSPA miss distance versus time for the proposed and standard multi-Bernoulli filters.

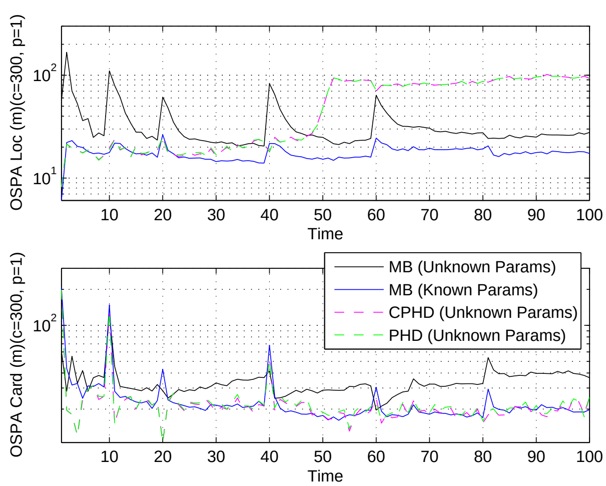

Fig. 6. OSPA localization and cardinality components versus time for the proposed and standard multi-Bernoulli filters.

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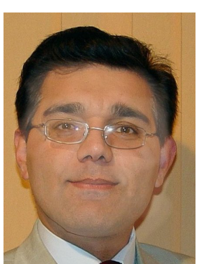

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